

Using Guided Math to Strengthen Students' Math Learning

Grades 3-6

PD RESOURCE KIT



Bureau of Education & Research

**Using Guided Math to Strengthen
Students' Math Learning
Grades 3-6**

PD Resource Kit

RESOURCE GUIDE

By Barbara Blanke, PhD
and Mona Roach, PhD



Bureau of Education & Research

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The purpose of this program is to demonstrate the power of guided math to enhance math learning for all students in intermediate grade classrooms. Included is a brief overview of the conditions essential to implementing guided math with grades 3-6 students. Participants will also see lessons that illustrate the key characteristics of guided math, and, as a bonus, model lessons to demonstrate how guided math works with intermediate grade students.

We have designed this program to be used in a variety of ways. PD trainers may use the video program and *Resource Guide* to support sessions with groups of any size. Individuals and small groups interested in self-study will also find the video footage and *Resource Guide* helpful.

This PD Resource Kit contains a comprehensive Resource Guide and DVD with video clips for use in PD sessions. We have designed each PD Resource Kit to be used by:

- PD facilitators to support PD sessions with groups of any size
- Individuals and small groups interested in self-study

The Resource Guide contains suggestions for utilizing the PD Resource Kit and a variety of print resources that may be reproduced for use by participants in their own classrooms.

Sincerely,

Mona Roach, PhD
Media Training Manager

ABOUT THE TRAINER . . .

BARBARA BLANKE is an outstanding math educator, author and national presenter with over 22 years of classroom teaching experience. She is known for her creativity, enthusiasm and practical ideas for teaching mathematics. Barbara's special passion is using small guided math groups to address the wide range of math skills and understandings of children in any given classroom, ranging from students who are gifted in math to those who struggle with basic math skills. Her ideas for creating, implementing and managing guided math groups are inspirational and work well with any math curriculum. Barbara is the author of *Bridges in Mathematics*, *Number Corner, Grade 3* and continues to develop and hone effective guided math strategies as an educational consultant for school districts throughout the United States.

ABOUT THE BUREAU . . .

BUREAU OF EDUCATION & RESEARCH is North America's leading provider of PD events and resources for professional educators. Founded in 1976, the Bureau provides national and regional PD programs across the United States and Canada. The Bureau also offers additional high-quality PD resources:

PD Resource Kits contain outstanding video clips and supporting print resources designed for schools and districts to use to conduct their own PD sessions

Online PD Courses contain outstanding video clips and supporting print resources designed for individuals and groups of educators to learn at their own convenience

On-Site PD Services enable schools and districts to bring outstanding BER Trainers to their own sites to facilitate customized professional development

National Train-the-Trainer Programs on cutting-edge topics enable schools and districts to train their own staff members who then conduct site-based professional development sessions

For further information about Bureau PD programs and resources, please contact us: www.ber.org or toll free (800) 735-3503.

Using Guided Math to Strengthen Students' Math Learning Grades 3-6

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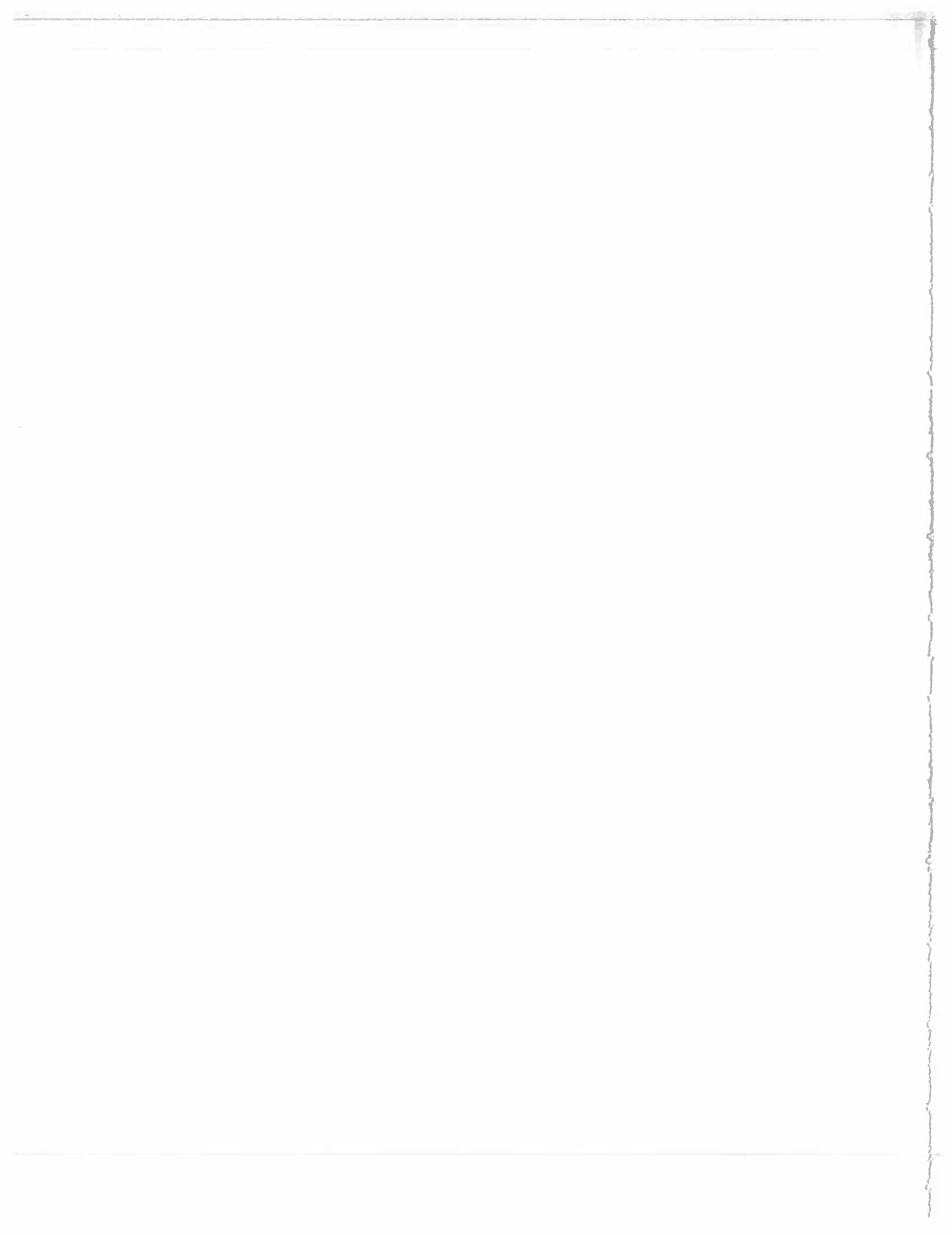
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PROGRAM GUIDE



Bureau of Education & Research



Suggestions for PD Trainers

General Information

This section of the *Resource Guide* is designed for those who will be presenting this material in a workshop format. If you are viewing the program by yourself or with a small group, please turn to page 11 for suggestions appropriate to your needs.

Video can be a powerful medium for demonstrating effective teaching practices and providing opportunities for discussion. This program offers facilitators a variety of options:

Option One: Select PLAY ALL on the main DVD menu. This selection enables you to control the video playback for the entire program, stopping when desired to ask questions, hold a discussion, and use the related print resources. You will also find this option helpful when previewing the program before a staff development session.

Option Two: Select a topic from the TOPIC MENU. The selection will either take you directly to video footage or bring up a more detailed Sub-Topic menu. Selecting PLAY ALL on the Sub-Topic menu will play the entire topic section. You can stop at any point for questions and discussion.

Option Three: SUB-TOPIC MENUS enable you to select specific segments within a topic. This is an excellent option when you want to review a particular aspect of the topic or wish to focus on just one lesson or concept.

No matter which option you choose, as you guide participants through this program, the most effective strategy is to show a segment, then stop for discussion and/or an opportunity to read the related print resources. At times, you may want to show viewers a particular segment more than once to clarify some finer points of the teaching. We encourage you to tailor your sessions to match the expertise and experience of your participants.

Overview of the Program

Using Guided Math to Strengthen Students' Math Learning, Grades 3-6 is a 65 minute video program designed to help intermediate grade classroom teachers implement guided math as an essential component of their mathematics program.

The first section of the program provides an overview of how to set the stage for successful implementation of guided math instruction. 9:18

The second section of the program focuses on five key characteristics of guided math lessons. 24:28

In the final portion of the program, you will see three short model lessons to demonstrate how guided math works with intermediate grade students. 29:24

Scheduling Suggestion

This program is not designed to be shown straight through. The program is ideal for a series of short training sessions. If you choose to show the entire program in a single session, the most effective approach is to show a section of the video, use the questions in the *Suggestions for PD Trainers* section of this guide to facilitate a discussion, and provide handouts from the *Print Resources*.

Equipment/Materials Needed for Training

DVD player, monitor

DVD

Handouts for participants

Chart paper or whiteboard, markers

Optional:

- LCD projector
- transparencies of training resource pages

Note-Taking Guide

A note-taking guide for the entire program is located on pages 21-31. The divisions and headings match the graphics in the video.

Discussion Questions

Discussion questions are provided to encourage participants to discuss, clarify, and consider the implications of implementing guided math with their own students.

The Training Sessions

Before Viewing the Program

To provide participants with a framework for watching the video program, distribute *Understanding Guided Math*, page 37 in the Resource Guide. After giving participants time to read the handout, ask them to generate a list of questions about guided math to keep in mind as they watch the video. You may want to return to the list at the end of the session to discuss which questions were addressed and to bring up any concerns or questions that may still exist.

INTRODUCTION AND SETTING THE STAGE FOR GUIDED MATH SUCCESS 9:18

This section of the program includes an introduction and a brief overview of key practices that set the stage for effectively implementing guided math:

- *Whole Class Instruction*
- *Independent Practice*

- *Flexible Grouping*
- *Forming Guided Math Groups*
- *Variety of Materials and Strategies*

Distribute the *Note-Taking Guide*, page 21.

Show the video.

By stopping the video after each segment and using the questions below, you will provide participants with valuable opportunities to process and discuss how to effectively incorporate key practices that set the stage for guided math into their math instruction. It is also an opportune time for experienced teachers to share ideas and suggestions for how they implement small group math instruction in their classrooms.

Whole Class Instruction 1:33

Discussion Questions

- ✓ What scheduling adjustments would you need to make in order to incorporate guided math as part of your math program? What changes need to occur at each grade level and/or school wide? What are some of the challenges you face in adjusting your math instruction?
- ✓ Why is active participation during whole class instruction critical to students' learning?
- ✓ In addition to brief partner sharing, what are some other ways to keep students engaged during whole class instruction?

Independent Practice 1:41

Discussion Questions

- ✓ What types of independent math practice activities are most effective for intermediate grade students?
- ✓ If you don't currently incorporate independent math practice, what materials and activities are available for you to start providing a few options for students that will enhance their understanding of grade level math concepts and skills?
- ✓ What routines and skills must be taught in order to effectively implement independent practice activities?
- ✓ What behavior guidelines or classroom agreements are in place that will enable you to work with small groups during math time?
- ✓ In your classroom, how do students get help during independent practice time when you are working with other students?

Flexible Grouping 1:00

Discussion Questions

- ✓ Why is flexible grouping critical to the effectiveness of guided math?
- ✓ In what ways does incorporating whole class instruction, independent practice and small guided math groups enable teachers to better meet students' needs?

Forming Guided Math Groups 1:19

Discussion Questions

- ✓ What are some strategies for forming groups and keeping track of individual student progress?
- ✓ What math assessments do you currently use?
- ✓ How do you use assessment data to make decisions about math instruction and materials?
- ✓ How do you fit assessment into classroom instructional time?
- ✓ How do structured and informal assessments combine to inform teachers' decisions when forming guided math groups?

Variety of Materials and Strategies 1:13

Discussion Questions

- ✓ Why is it critical to incorporate varied materials and techniques when working with intermediate grade students?
- ✓ Why do intermediate grade struggling learners often need manipulatives and other visual/concrete support to develop math concepts?

Supporting Print Materials

Note-Taking Guide (page 21)

What is Guided Math? (page 33)

Understanding Guided Math (page 37)

Math Assessment Strategies (page 39)

Observational Tools for Math Assessment (pages 41-45)

Productive Independent Math Practice Activities (pages 47-53)

References and Resources (pages 141-147)

KEY CHARACTERISTICS OF GUIDED MATH LESSONS 24:28

This section of the program demonstrates five key characteristics of guided math lessons:

- *Lessons are short and focused on targeted skills*
- *Lessons start with a brief introduction to the task*
- *Students work individually as teachers observe and coach*
- *Students share their thinking with the group*
- *Lessons end in a timely manner*

At first glance, guided math may seem similar to typical small group instruction; however, it has some unique characteristics that make it much more effective and practical.

- Guided math lessons start with what students know and move them to new learning; because math skills and concepts build sequentially, this technique helps avoid gaps in students' math development.
- Lessons are brief and most of the time is dedicated to students working independently with at-the-moment teacher coaching and prompting.
- Teachers must keep the pace of the lesson moving in order to include student sharing; teachers may end an activity before "mastery" of a concept or skill is reached; students at this point of "partial thinking" are primed for learning and respond well to additional teacher coaching.

Directions to Trainers

Distribute the *Note-Taking Guide*, page 23.

Show the video.

Using the Sub-Topic Menu option on the DVD enables you to show a segment and then use the questions below to engage participants in discussions to increase their understanding of and their ability to implement guided math. It is highly recommended to have participants re-watch the segment *Students Work Independently as Teachers Observe and Coach* in order to more fully understand the finer points of effective coaching during guided math.

Lessons are short and focused on targeted skills 1:12

Discussion Questions

- ✓ What are some benefits to keeping guided math lessons short and focused?
- ✓ What lessons or activities would work best in a guided math time frame?

Lessons start with a brief introduction to the task 6:10

Discussion Questions

- ✓ What is the purpose of a guided math introduction?

- ✓ Why keep introductions brief? What are some steps to take to help students who don't understand the task? If students are having difficulty understanding the directions, what are some options?

Students work independently as teachers observe and coach 9:39

Discussion Questions

- ✓ Why might students working independently with teacher coaching be called the most vital part of a guided math lesson?
- ✓ Why is it important to wait until students have started to work independently before starting to coach or prompt?
- ✓ Why is questioning such a critical strategy for enhancing students' math learning?
- ✓ Notice the amount and types of teacher talk during guided math lessons. How does this compare to typical small group math instruction?

Students share their thinking with the group 2:32

Discussion Questions

- ✓ Why is it critical to be diligent about having every student share his/her thinking out loud during guided math lessons?
- ✓ What are some ways to manage student sharing out loud—for both children who have a lot to say and for those who have a hard time articulating their ideas?

Lessons end in a timely manner 4:10

Discussion Questions

- ✓ What lesson closing options appeal to you? How much does the strategy you use depend upon the lesson and students?
- ✓ How is guided math similar to guided reading? In what ways is it different?
- ✓ What makes guided math different from traditional math groups?

Supporting Print Materials

Note-Taking Guide (page 23)

Conducting a Guided Math Session (page 55)

The Art of Questioning in Mathematics (page 57)

Genuine Questions (page 59)

Larger Numbers on a Line (pages 61-67)

Multiplication Challenge (pages 69-75)

Fractions of a Foot (pages 77-87)

Introduction to Egg Carton Fractions (pages 89-101)

Egg Carton Fractions Record Sheet (page 103)

Fractions on a Clock (pages 105-117)

Fact Fluency with 5's (pages 119-129)

Problem-Solving Guided Math Lessons (page 131)

Football Score math problem (page 133)

References and Resources (pages 141-147)

MODEL LESSONS AND CLOSE

Each model lesson provides a real-time example of guided math. Unplanned and unrehearsed, each lesson gives a snapshot of the brief, yet powerful nature of guided math for coaching students at just the right moment to scaffold their learning.

MODEL LESSON 1 10:44

The purpose of this guided math lesson is to extend students' math problem-solving skills and encourage independent thinking. Of particular note is how Barbara Blanke uses wait time, questions and strategic suggestions to support students as they work.

MODEL LESSON 2 4:14

The purpose of this lesson is to provide follow-up from the first lesson and enable students to share their independent thinking. Of particular note is how Barbara Blanke validates students' ideas and fosters new mathematical understandings.

MODEL LESSON 3 14:26

The purpose of this lesson is to have students look for and use mathematical patterns and encourage their use of logical reasoning. Of particular note is how Barbara Blanke effectively maintains lesson pacing, keeps students engaged, and skillfully uses manipulatives to scaffold their learning.

Directions to PD Trainers

Preview the model lessons to help you guide participants to a deeper understanding of guided math. Becoming familiar with each lesson will enable you to pose questions and prompts to reveal the elements of guided math and how they work together to provide focused learning opportunities for students.

Have participants read or re-read *What is Guided Math?* (page 33) and/or *Understanding Guided Math* (page 37) and then briefly discuss how guided math compares to guided reading and to more typical math group instruction to set the stage for viewing the model lessons.

Distribute the corresponding *Note-Taking Guide* (pages 25-31) for each model lesson. The note-taking guides are intended to help participants look more closely at each lesson to better understand how guided math works. Each page is organized into two columns. The column Delving into the Guided Math Lesson lists several attributes or aspects of a guided math lesson.

As participants watch each model lesson, they record their observations in the Evidence column. It is quite helpful to watch the lesson again, giving participants time to add more information to their handouts and discuss their findings.

Supporting Print Materials

Note-Taking Guides (pages 25-31)

Hannah's Problem (page 135)

Stacking Dice: Is it Magical or Mathematical (pages 137-139)

References and Resources (pages 141-147)

Suggestions for Self-Study

General Information

This section of the *Resource Guide* is designed for individuals and small groups. If you are responsible for leading a group through this material, please turn back to *Suggestions for PD Trainers*, page 3.

Video instruction provides a practical and efficient way to observe effective teaching strategies and engage in thoughtful reflection. We encourage you to watch a segment, stop the video, reflect, and take full advantage of the related print resources. You will find suggestions for making the most of your sessions in this portion of the *Resource Guide*.

Overview of the Program

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The first section of the program provides an overview of how to set the stage for successful implementation of guided math instruction. 9:18

The second section of the program focuses on five key characteristics of guided math lessons. 24:28

In the final portion of the program, you will see three short model lessons to demonstrate how guided math works with intermediate grade students. 29:24

Scheduling Suggestion

Whether you choose to view the entire program in one session or view sections of the program over two or more study sessions, **this program is not designed to be viewed straight through.** Your learning experience will be much richer if you take the time to stop the video after each section, reflect, and read the additional information contained in this *Resource Guide*.

Equipment/Materials Needed

DVD player, monitor

DVD

Print Resources (pages 21-147)

Reflection Questions

Reflection questions are provided for your sessions. It is recommended that you view the segment and then refer to the questions for thoughtful reflection. Please keep in mind that this guide contains print explanations of the strategies as well as other related material that can inform and enrich your learning.

Note-Taking Guide

A note-taking guide for the entire program is located on pages 21-31. The divisions and headings match the graphics in the video.

Your Session

INTRODUCTION AND SETTING THE STAGE FOR GUIDED MATH SUCCESS 9:18

This section of the program includes an introduction and a brief overview of key practices that set the stage for effectively implementing guided math:

- *Whole Class Instruction*
- *Independent Practice*
- *Flexible Grouping*
- *Forming Guided Math Groups*
- *Variety of Materials and Strategies*

Turn to the *Note-Taking Guide*, page 21.

Watch each segment of the video. As you watch, consider your own classroom and students in light of the suggestions in the program. Stopping the video after each segment and reflecting on the questions below is designed to enhance your learning and implementation of guided math.

Whole Class Instruction 1:33

Reflection Questions

- ✓ What scheduling adjustments would you need to make in order to incorporate guided math as part of your math program? What changes need to occur at each grade level and/or school wide? What are some of the challenges you face in adjusting your math instruction?
- ✓ Why is active participation during whole class instruction critical to students' learning?
- ✓ In addition to brief partner sharing, what are some other ways to keep students engaged during whole class instruction?

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Reflection Questions

- ✓ What types of independent math practice activities are most effective for intermediate grade students?
- ✓ If you don't currently incorporate independent math practice, what materials and activities are available for you to start providing a few options for students that will enhance their understanding of grade level math concepts and skills?
- ✓ What routines and skills must be taught in order to effectively implement independent practice activities?
- ✓ What behavior guidelines or classroom agreements are in place that will enable you to work with small groups during math time?
- ✓ In your classroom, how do students get help during independent practice time when you are working with other students?

Flexible Grouping 1:00

Reflection Questions

- ✓ Why is flexible grouping critical to the effectiveness of guided math?
- ✓ In what ways does incorporating whole class instruction, independent practice and small guided math groups enable teachers to better meet students' needs?

Forming Guided Math Groups 1:19

Reflection Questions

- ✓ What are some strategies for forming groups and keeping track of individual student progress?
- ✓ What math assessments do you currently use?
- ✓ How do you use assessment data to make decisions about math instruction and materials?
- ✓ How do you fit assessment into classroom instructional time?
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Watch the video.

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Reflection Questions

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Supporting Print Materials

- Note-Taking Guide* (page 23)
- Conducting a Guided Math Session* (page 55)
- The Art of Questioning in Mathematics* (page 57)
- Genuine Questions* (page 59)
- Larger Numbers on a Line* (pages 61-67)
- Multiplication Challenge* (pages 69-75)
- Fractions of a Foot* (pages 77-87)
- Introduction to Egg Carton Fractions* (pages 89-101)
- Egg Carton Fractions Record Sheet* (page 103)
- Fractions on a Clock* (pages 105-117)
- Fact Fluency with 5's* (pages 119-129)
- Problem-Solving Guided Math Lessons* (page 131)
- Football Score math problem* (page 133)
- References and Resources* (pages 141-147)

MODEL LESSONS AND CLOSE

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MODEL LESSON 3 14:26

The purpose of this lesson is to have students look for and use mathematical patterns and encourage their use of logical reasoning. Of particular note is how Barbara Blanke effectively maintains lesson pacing, keeps students engaged, and skillfully uses manipulatives to scaffold their learning.

Turn to the *Note-Taking Guide* for each model lesson (pages 25-31.) These handouts are intended to help you look more closely at each lesson to better understand how guided math works. Each page is organized into two columns. The column Delving into the Guided Math Lesson lists several attributes or aspects of a guided math lesson.

After watching a model lesson, you may want record your observations in the Evidence column. It is quite helpful to watch the lessons again, paying more attention to specific elements of guided math.

Supporting Print Materials

Note-Taking Guides (pages 25-31)

Hannah's Problem (page 135)

Stacking Dice: Is it Magical or Mathematical (pages 137-139)

References and Resources (pages 141-147)



PRINT RESOURCES



Bureau of Education & Research

Note-Taking Guide

Using Guided Math to Strengthen Students' Math Learning, Grades K-2

Introduction

Conditions for Guided Math Success

Whole Class Instruction

Ongoing Student Assessment

Independent Math Activities

Management Routines

- Classroom Agreements

Flexible Grouping

Creative Use of Space

Variety of Materials and Strategies

Key Characteristics of Guided Math Lessons

Short Lessons

Introduce the Task

Students Work Individually with Teacher Coaching

- Start by observing students
- Strategic coaching
- Highly engaging activities

Students Share Their Thinking

Lessons End in a Timely Manner

Model Lesson One

Delving into the Guided Math Lessons	Evidence
<p>Targeted Math Concept/Skill</p> <ul style="list-style-type: none"> • extend students' math problem-solving skills • encourage independent thinking 	
<p>Focus of Teacher Coaching</p> <ul style="list-style-type: none"> • wait time • questions • strategic suggestions, prompts 	
<p>Student's response to teacher coaching</p> <ul style="list-style-type: none"> • Student A • Student B • Student C 	
<p>Lesson pacing <i>Total Lesson: 10:44</i></p> <p>Teacher introduction 2:31</p> <p>Students work independently 5:10</p> <p>Students explain their thinking 2:21</p> <p>Lesson ends 0:32</p>	
<p>How did students respond to being stopped to discuss their "partial thinking"?</p>	

How did Barbara handle the girl who was focusing on common multiples of 4 and 5?

What might you do to capitalize on the student's discovery at a later time?

Were there any teachable moments that you could have used to enhance students' understanding of this math problem? If so, what would you have done?

Model Lesson Two

Delving into the Guided Math Lessons	Evidence
<p>Targeted Math Concept/Skill</p> <ul style="list-style-type: none"> • follow-up from the first lesson • enable students to share their independent thinking 	
<p>Focus of Teacher Coaching</p> <ul style="list-style-type: none"> • validate students' ideas • foster new mathematical understanding 	
<p>Student understanding of math concepts</p> <ul style="list-style-type: none"> • Student A • Student B 	
<p>Lesson pacing <i>Total Lesson: 14:14</i></p> <p>Teacher introduction <i>0:40</i></p> <p>Students explain their thinking <i>3:19</i></p> <p>Lesson ends <i>0:15</i></p>	

How did Barbara manage the student sharing to give each student an equal chance to describe his or her thinking?

What's the next instructional action for these students based on this guided math lesson?

Model Lesson Three

Delving into the Guided Math Lessons	Evidence
<p>Targeted Math Concept/Skill</p> <ul style="list-style-type: none"> • students look for and use mathematical patterns • develop their use of logical reasoning 	
<p>Focus of Teacher Coaching</p> <ul style="list-style-type: none"> • maintains lesson pacing • keeps students engaged • skillfully uses manipulatives to scaffold students' learning • Notice Barbara's artful use of questions, summary statements, and re-stating students' ideas 	
<p>Student understanding of math concepts</p> <p>Student A</p> <p>Student B</p> <p>Student C</p> <p>Student D</p>	
<p>Lesson pacing <i>Total Lesson: 14:26</i></p> <p>Teacher introduction 5:00</p> <p>Students work independently 4:00</p> <p>Students explain their thinking 5:00</p> <p>Lesson ends 0:26</p>	

Model Lesson Three, *continued* ...

In what ways does this guided math lesson differ from model lessons 1 and 2?

What are the similarities?

What are some specific actions Barbara took to encourage the boy who was struggling with the concept of opposite sides of a die equaling 7?

Were there any teachable moments that you could have used to enhance students' understanding of this math problem? If so, what might you have done?

What is Guided Math?

"All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding. There is no conflict between equity and excellence.

NCTM, 2000

Guided math is a structured, practical way of matching math instruction to the diverse individual learners in the classroom. This approach is based on the belief that every child is capable of learning and understanding mathematics and recognizes that students learn to do mathematics at varying rates of development and over time.

Guided math provides each child with an opportunity to participate in a small-group setting where the teacher can tailor mathematical learning experiences to meet all students' needs. Through modeling, prompting and asking genuine questions, the teacher guides and coaches students to think about math learning and develop fluent, efficient strategies that make sense. These small group sessions provide personalized practice opportunities for the students. The teacher's instruction is guided by her careful observation and assessment of each student's mathematical thinking and use of strategies.

The goal of guided math is to assist students in using reasoning and logic, as well as basic skills, to solve mathematical problems independently and accurately. This sense-making process is the foundation for being a lifetime mathematician. Each time a student participates in a guided math lesson, the teacher provides support for the child to apply known strategies and skills and coaches him or her to new math understandings. In time, students will be able to extend their repertoire of problem-solving strategies and skills to successfully solve new problems independently, becoming autonomous learners.

Teachers rely on assessments to form groups of students with similar needs. As students' needs change so does the composition of the guided math groups. Ideally, teachers conduct guided math sessions two or three times a week, choosing to work with low progress students as often as possible.

Student assessment also provides the basis for selecting activities that provide appropriate levels of challenge to each learner in the group. Guided math sessions are brief and targeted, providing 5-15 minutes of uninterrupted time for students to focus on a math problem, game, activity or task with teacher coaching. Guided math sessions can be conducted at a table, on the floor, or at a small group of student desks. Teachers will want to have the necessary materials and tools for each lesson on hand and easily accessible to make the most of their time with each group.

To start the lesson, the teacher quickly introduces the task making sure to model or introduce new ideas or tricky vocabulary. As individual students engage in the activity, teachers ask thought-provoking questions to guide and prompt students as necessary throughout the session. Jotting down brief observational notes about individual students as they work will provide teachers with important information about each group member's understanding of mathematical concepts.

After students have worked individually on a problem or task for a few minutes, the teacher brings the group together to briefly share their strategies and math thinking. In order to ensure time for group sharing, teachers may have students stop before they finish a problem.

Asking students to share their “partial thinking” provides opportunities to assess their understanding as well as scaffold their learning.

To close a guided math lesson, teachers may assign students to continue working on the math task or activity as part of their independent practice. Students may also be given a more challenging problem to work on independently, then share their thinking at another guided math session. Teachers may also simply end the lesson with a quick summary statement. When students leave a guided math session, they need to be clear about what to do next to help ensure a smooth transition.

Understanding Guided Math

"The aim of guided math is to develop autonomous learners who question, consider alternatives, and make informed choices as they seek to make sense of mathematics."

Barbara Blanke, 2011

Similar to guided reading, guided math is a teaching approach designed to help individual students learn how to process a variety of increasingly challenging mathematical problems and/or skills with deep understanding while at the same time developing computational fluency. Like guided reading, guided math occurs in a small-group context because the small group allows for interaction among the learners and with the teacher. The teacher selects and introduces a math problem or task and supports students with coaching and prompting while they independently work on the math task. The lesson engages students in discourse and/or discussions about math concepts and skills as the teacher makes explicit teaching points. The groups are flexible and change depending on individual student learning needs.

Guided math represents a different approach than that of traditional math groups.

Guided Math Groups	Traditional Math Groups
Selection of the mathematical problem, concept, or skill is carefully planned by the teacher to match the students' instructional needs and/or interests.	Selection of the math lesson is determined by the textbook or the math program, regardless of students' instructional needs and/or interest.
For the majority of the time, students work independently on a math task and the teacher coaches and prompts with genuine questions.	Students are directly taught steps to follow to get to the correct answer of a problem.
Students strategize in a variety of ways to understand the math.	Worksheets are typically used to drill math skills.
Problem-based lessons are often used to deepen mathematical understanding.	The math textbook guides all instruction for mathematics in the small group.
The focus of the session is on skills and strategies for autonomous learning of mathematical concepts & skills.	The focus is on skills to procedurally get to the correct answer.
Instruction is based on what students know and are able to do as identified through continuous assessment with a variety of assessment methods and tools.	Instruction is focused on a step-by-step progression of skills in the textbook as measured by an end of the unit post-test.
Flexible math groups change based on ongoing assessment as skills or strategies are learned.	Fixed, leveled groups usually remain together during the entire unit or year of instruction.
Math is connected to real world contexts and integrated in other subject areas.	Mathematics tends to be treated as a separate subject.
Lessons are brief and paced to ensure time for all group members to share their thinking	Lessons are pre-determined and teacher led; may or may not provide time for all students to talk

Math Assessment Strategies

Assessment is “the process of gathering evidence about a student’s knowledge of, ability to use, and disposition toward mathematics and of making inference from that evidence for a variety of purposes”. (NCTM, 1995, p.3)

How do we assess for guided math?

Monitoring Student Progress

Assessment activities should provide both teachers and students with productive, ongoing feedback regarding their progress toward specific lesson objectives and the long-term goal of developing students’ deep understanding of mathematical concepts and skills.

Evaluation Options

The teacher in a guided math classroom will collect evidence from various forms of assessments. These can include: pretests, post-tests, quizzes, observations, student work samples, interviews, and student discussions. Evaluations should reflect performance criteria about what students understand and are able to use to solve problems.

Journaling and Writing

Writing and journaling provide both learning and assessment opportunities in the mathematics classroom, especially conceptual understanding and problem solving. Journals are a way to make written communication a regular part of learning mathematics, especially with intermediate grade students. Using a daily math journal is an effective and efficient way to encourage students to share their thinking, solve problems, and ask questions.

Pencil and Paper Tests

Written tests will always be a part of guided math assessments and evaluations. However, a test does not have to be a collection of low-level skills that are simple to grade. Involving students in higher-level thinking questions will give the students and teachers more information about their level of understanding.

Interviews

Interviews are rich assessments that provide evidence of students’ misunderstandings and their understandings of the concept being learned. Although time intensive, interviews offer explanations into students’ current thinking about how they got to a correct answer or where they are experiencing problems.

Making Instructional Decisions

Teachers need to continually reflect on what they see students doing in every lesson and the thinking that accompanies those actions. Teachers are looking for evidence of academic growth to help them formulate the next best steps to assist students to develop ideas rather than remediate their teaching after the fact.

Observational Tools for Formative Assessment in Guided Math

When teachers listen attentively and ask genuine questions they can learn useful bits of information about their students on a daily basis. It is important to have a systematic approach to recording these observations. Systematic observational data can contribute to evaluative data and assist teachers in planning next guided math lessons, providing feedback to students about their learning, sharing students' academic growth with parents during conferences, and contributing data for grades. There are numerous methods that teachers can use to collect this type of data.

Recording Anecdotal Notes

Writing short notes during or immediately after a guided math lesson in a brief narrative style is a great way to document student growth. One option is to use a clipboard or file folder and an index card for each student. Tape the top of an index card to the clipboard or file folder so that the bottom of the card lines up with the bottom edge of the folder. Tape the next index card on the clipboard or folder so that about an $\frac{1}{2}$ inch of the first index card is showing. Continue taping index cards, staggering the bottoms of each card. Write each student's name on the edge of each card that is showing. As you observe students, flip to their card and jot down quick notes. Date the card every time you record an observation. If you color code your cards in groupings of 5 or 6 you can collect data on different students on different days. For example, green cards are Monday observations; red cards are Tuesday observations, etc.

An alternative to cards is to record notes on peel off labels pre-printed with student names. When labels become full, remove and adhere them to students' portfolios throughout the year.

A third way to keep track of observational notes is to use a spiral notebook and tab every 8-10 pages with each student's name. You can then use the notebook to date and record your observations throughout the year and document growth over time.

Using a Rubric as an Observational Tool

Teachers can use three or four point rubrics to create a generic reusable form to keep track of student observation during guided math lessons. Place the rubric descriptors and content specific indicators on the left and jot down the names of students who are demonstrating that rubric level in the column on the right. A quick comment can be added to any student's name. This form can be used over several days of instruction on a single concept/topic. The rubric is also helpful in planning for your guided math groups.

For intermediate grade students, rubrics are ideal for self-assessing their own mathematical learning progress. The same forms can be slightly altered to invite students to assess their own strengths and areas of challenge.

Observation Rubric Date:	Concept/Standards:
Exceeds Expectations Shows a clear understanding. Can communicate the concept in various ways. Can use the concept/idea/skill without prompting.	
Meets Expectations Understands concept or demonstrates a developing understanding. Uses provided models.	
Moving Toward Expectations Some confusion or misunderstanding of the concept/skill. Can use models/idea only with support.	

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Moving Toward Expectations Some confusion or misunderstanding of the concept/skill. Can use models/idea only with support.	

Individual Student Checklists

Creating a checklist with multiple concepts and skills for each individual student is another way to record observations. Sometimes these can be printed on a sheet of labels or cards to save time. Some teachers prefer a whole sheet of paper to keep more anecdotal notes. Once a template is created, it is easy to adjust to new concepts and skills. A place for teacher observations and comments should be part of the checklist.

Student Name:	Emerging	Meets Expectation	Exceeds Expectation	Comments
Concept				
1. Skill				
2. Skill				
3. Skill				
4. Skill				
Problem Solving Skills <ul style="list-style-type: none"> • Comprehends problem before beginning to solve • Takes risks and tries a new strategy • Can justify or prove the result/answer 				

Productive Independent Math Practice Activities

One of the keys to guided math is that the students who are not meeting with the teacher are productively engaged in rich, independent math activities that provide practice of basic skills and problem-solving tasks. Math practice activities and centers vitalize classrooms, even though they do take a bit of work, planning, and organization to be successful. Effective independent activities motivate and energize students to learn. Math centers and independent practice activities are dynamic and constantly evolving to meet students' needs and match teachers' growing understanding of the vital role of practice in effective math instruction.

Building a Community of Math Learners

It is important to set the stage for independent math practice and center activities by engaging students in a discussion about becoming a community of math learners who support each other's learning. When students are involved in creating the guidelines, it is much easier for them to be accountable for their behavior. Establishing shared expectations for behavior during math time keeps students productively engaged in independent practice while the teacher works with small guided math groups.

Begin by brainstorming a list of ideas that will eventually become Community Agreements. Focus on the positive language of "What we agree" vs. "What we don't do" when creating your agreements. Your final list might look something like this:

How to Be a Community of Math Learners:

- Listen to others and respect their thinking
- Ask thoughtful questions
- Disagree with others in a respectful way
- Volunteer your ideas in group discussions
- Don't be afraid to struggle with challenging ideas and problems
- It's okay to be confused
- Enjoy discovering new things about math

Occasionally revisit Community Agreements before students work independently or with partners and celebrate the successes when students abide by the agreed-upon behaviors.

Managing Math Learning Centers

Math centers and independent practice activities do not have to be permanent locations in your classroom. Basically, a math "center" consists of activities with a shared focus. Center activities can be kept in boxes, tubs and bags. They can be stored in cupboards or on shelves. During independent math practice, students work at their desks, at tables and counters, on the floor, or in areas specifically designated for use during that time.

There are four keys to managing math learning centers and independent practice activities:

1. Students must be involved in rich, independent activities that involve practice and problem solving.
2. Effective centers and activities are able to be differentiated and adjusted to maintain student engagement for 4 – 6 weeks.
3. When students finish one center or activity, they move to the next center by choice or by assignment based on academic need.
4. Make sure students are clear about how many students can be engaged in a learning center or activity at a time.

Grouping for Instructional Success

Research has shown that centers and independent activities work best when children with varying capabilities engage in learning together. Heterogeneous (mixed-ability) groups lead to higher-quality experiences for all children. It is important to include varied topics and ways to demonstrate knowledge so that different students can step forward at different times to take on the responsibility of initiating discussions, explaining, modeling, and problem solving. Teachers need to trust that children's different kinds of knowledge and different ways of knowing will prompt them into meaningful collaboration and new ways of thinking about the problems at hand.

Rotational or Choice Models

Math learning centers and independent practice can be implemented in various ways. Most teachers think of rotational systems when they think about centers. This is one approach. Another way, which is a personal preference of Barbara Blanke, is an organized system that supports a student choice model.

Within a reasonable framework, students need to be able to make good choices about their own learning. Providing 6-8 math learning centers for students to choose from will allow students to work at their individualized pace, choose the order in which they complete tasks and the frequency with which they return to certain activities. Students can choose a learning partner(s) and level of difficulty of a task for additional choice options. Teachers can learn about students' learning preferences by providing different learning activities from which to choose. Through careful planning and selection of activities, teachers can set up 6-8 activities and use them for 4-6 weeks of instructional time. This reduces the amount of time that a teacher spends creating rotational center activities on a weekly basis. Students should complete all learning centers at least once and then they may choose to continue to practice at the ones that they enjoy most. Some learning center activities require students to work independently, others require partners, and some centers may even engage small groups in an activity.

The best systems are easy to understand, easy to explain, and easy to monitor. And keep in mind that you can always adapt your system to better address problem areas or meet changing needs. Think of your management plan as a work in progress and don't hesitate to refine it over time.

Teacher as Facilitator

Engaging the rest of the class in independent math activities or centers enables teachers to work with small groups of students for guided math. In order to maintain the integrity of independent math activities and centers and ensure students are being productive, teachers periodically check on students between guided math groups. Teachers may also choose to rotate around the classroom and "kidwatch" or observe how each child is doing; in a whole group instructional method, this wouldn't be possible. Because the teacher watches each child closely, she is then able to assess her students' individual needs and plan accordingly.

Obtaining and Organizing Materials

Because math learning centers and independent activities require access to an abundance of materials and tools, keeping them organized is essential for easy retrieval. An effective basic storage system should include boxes, file folders, and sturdy containers. Each container should be labeled, with an attached list of its contents, and their spots should be labeled on the shelf where they should be returned. This will ensure that students can retrieve and return all materials in a time efficient manner. Most standards-based mathematics curricula provide basic manipulatives like pattern blocks, wooden cubes, connecting cubes, rulers, square tiles, etc. You may want to enlist your school community to donate supplies as well. Items should be organized and accessible at all times during math learning center time. Many teachers choose to label stacking drawers with picture labels of these materials for ease of storage space and student accessibility. Specific materials for each learning center can be run off and placed in a file folder in a tub, box, hanging file system or dishpan. Small, zip-top plastic bags are useful for organizing loose materials for learning stations. General materials like pattern blocks, cubes, rulers, calculators should be kept in easily accessible tubs/boxes/drawers so students can use these tools as needed. Incorporating different learning modalities into center activities helps accommodate the needs of all students.

Modeling and Introducing Activities

When introducing math center and independent practice activities, taking time to model activities helps better ensure student confidence and independence. As you plan for independent activities, take into consideration what can and should be modeled before students are expected to function independently. One option is to model activities during whole group instruction. You may also want to demonstrate centers and independent activities to small groups of students to explain more specific details.

Transitions

- Ineffective transitions during math centers and independent practice can lead to the loss of valuable learning time. Have a clear picture in your mind of how a smooth transition should look and sound. Make sure from the very beginning of the year you communicate your expectations for transitions, provide explicit directions, practice transitions with students, and actively identify what went well during transition time. Here are some suggestions for setting the stage for smooth transitions:
- Use a signal to quickly get students' attention
- If you have students who have difficulty meeting your expectations, partner them with another student or hold a goal planning session with that student.
- Give students the responsibility of being a learning center leader. These are the students who other students can get assistance from, while you are teaching your small groups or working with individual students.

Set-up and Clean-up

Students must be able to go to a learning center or secure the appropriate activities, determine what they are supposed to do, make sure they have the right materials, and begin work without teacher assistance. The most effective independent math activities and centers give students practice with math concepts and skills that have been taught in the whole group sessions and guided math. Use a verbal or nonverbal signal to indicate when independent practice time is over and students need to clean up (Ex: frog clicker, song, chime, word or phrase). Because centers incorporate a variety of materials and activities, teachers will want to teach and reinforce their expectations for putting away materials and reducing the amount of materials lost or damaged. If the classroom is well-organized, then set-up and clean-up are the responsibility of the students NOT the teacher.

Conducting a Guided Math Session

“Effective Mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.”

NCTM, 2000

Here is a proposed sequence for planning and teaching a guided math session:

Before:

- Teacher selects three to six students for the group and decides on the focus of the session based on her assessments of the students.
- Teacher chooses a mathematical activity or problem that will support the selected focus/Big Idea. She plans how to introduce the activity; how to model a strategy; chooses genuine questions to use to prompt the students’ thinking; and determines which students may need closer observation during the session.

During:

- Teacher introduces the problem/activity, often with the question: “What do you notice?” The following are other things the teacher may do or say:
 - What do you know about today’s problem/activity?
 - What does the problem tell us?
 - What do you think the problem is asking us to do?
 - What words are tricky? Anticipate new vocabulary that may challenge the students.
 - What do you predict will happen next?
 - What connections to other ideas to you see in today’s activity?
- Students solve the problem/begin the activity independently. The teacher observes the group and coaches individual students as needed, possibly focusing on one or two students.
- Teacher records her observations as anecdotal notes.

After:

- Teacher and students discuss the problem as a group to explore various strategies, and understandings of the problem-solving process. Emphasis is on the process.
- Students share strategies, partial thinking, and solutions.
- Teacher may clarify, re-teach, review skills or vocabulary used in the session.
- Teacher records observations and evaluates each student’s problem solving/basic skills demonstrated to begin to plan what needs to be learned next. Students may be involved with self-evaluating, especially if rubrics are used. This last step leads directly into the planning stage of the next guided math session.

The Art of Questioning in Mathematics

Questions to Help Students Work Together to Make Sense of Mathematics

- What do others think about what _____ said?
- Do you agree? Disagree? Why or why not?
- Does anyone have the same answer but a different way to explain it?
- Can you convince the rest of us?

Questions to Help Students Gain Confidence in Their Mathematical Thinking

- Why do you think that?
- How did you reach that conclusion?
- Does that make sense?
- Could you show us how you figured that out?

Questions to Help Students Reason Mathematically

- Does that always work? Why or why not?
- Can you think of another example?
- How could you prove that?

Questions to Help Students Conjecture, Invent, and Solve Problems

- What would happen if _____?
- Do you see a pattern?
- Can you predict the next one? the last one?
- What is alike and what is different about your method of solution and _____'s method?

Questions to Help Students Make Connection Between Mathematical Concepts

- How does that relate to _____ ?
- What have we learned before that was helpful in solving this problem?
- Can you give an example of _____ ?

Genuine Questions

When teachers ask students genuine questions, there are no right or wrong answers. The purpose of genuine questions is to uncover and discover what students are thinking. Following are some examples of genuine questions that can be adapted to any math topic or problem.

What do you think?

How would you explain _____ to a student who doesn't understand?

Could you explain _____ in another way?

How can you be sure that _____ ?

Can you explain your reasoning?

Can you draw a picture or build a model to illustrate _____ ?

How do you feel about _____ ?

Is there anything you don't understand about _____ ?

What is your prediction about what will happen?

What else would you like to know?

What do you plan to do next?

What is the most important idea or fact you learned while working on _____ ?

Why do you feel that way?

What were your first thoughts about _____ ?

What was/is the most challenging/easiest part of _____ for you?

What do you understand now that you didn't understand before?

What caused you to have a breakthrough in your understanding of _____ ?

What pictures do you have in your mind to help you think about _____ ?

How else do you think you could solve this problem?

How would you describe this problem in your own words?

What mathematical connections did you make?

What didn't work?

Are there any relationships in this problem that will always be present in similar problems?

What if _____ ?

What's your idea?

What were your thought processes while you worked on _____ ?

What do you wonder about regarding _____ ?

Where did you get "stuck" and what helped you get "unstuck?"

What mathematical insights did you have and what do you think prompted them?

What are your observations about _____ ?

Support Activity 8 ★ Instructional Considerations



SUPPORT ACTIVITY

Larger Numbers on a Line

Overview

Students take turns using the open number line to solve triple-digit subtraction problems. After they have both gone twice, they add their differences together. The player with the larger sum wins.

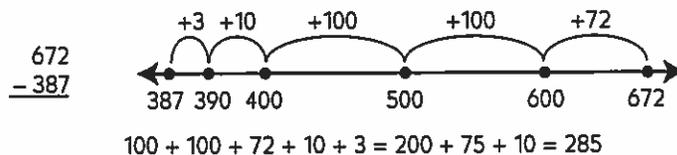
Skills & Concepts

- ★ locating and placing numbers on a number line
- ★ using models, pictures, and/or numbers to find the difference
- ★ identifying the operations needed for solving a problem
- ★ subtracting 3-digit numbers with and without regrouping using models and a variety of strategies

You'll need

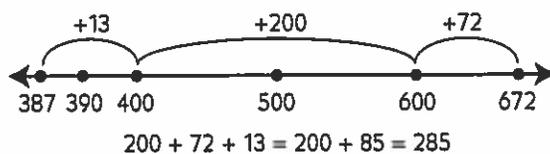
- ★ Instructions for Larger Numbers on a Line (Blackline S 8.2)
- ★ Larger Numbers on a Line Problem Cards (Blackline S 8.3, 1 copy cut apart and stored in an envelope or resealable plastic bag for each pair of players)
- ★ Larger Numbers on a Line Record Sheet (Blackline S 8.4, 1 copy run double-sided for each player)

Students who are not yet comfortable with the formal algorithms for addition and subtraction of larger numbers often find it helpful to use a number line when asked to perform these kinds of calculations. They can quickly sketch their line and count in chunks, confident that they have reached the correct answer. As they work, encourage them to look out for different landmark numbers and to use larger, more efficient chunks to add up from the smaller number to the larger number.



Mrs. Powell Now that you have this all written out, can you see another way to get from 387 to 672 with fewer, bigger jumps?

Jorge Well, I could have done one jump from 400 to 600. That would be 200. And now I guess I could see it's 13 from 387 to 400. I didn't think about that before.



Support Activity 8



SUPPORT ACTIVITY

Larger Numbers on a Line

You'll need

- ★ Instructions for Larger Numbers on a Line (Blackline S 8.2)
- ★ Larger Numbers on a Line Problem Cards (Blackline S 8.3, 1 copy cut apart and stored in an envelope or resealable plastic bag for each pair of players)
- ★ Larger Numbers on a Line Record Sheet (Blackline S 8.4, 1 copy run double-sided for each player)

Instructions for Larger Numbers on a Line

1 Write your name and the date at the top of a Larger Numbers on a Line Record Sheet.

2 Pick one problem card. Write the number of the problem on your record sheet.

3 If you need help reading the problem, ask another student to help you. Think carefully about what the problem is asking.

4 Use the open number line to show how you solved the problem.

5 Record your solution with a number sentence.

6 Take turns until you have both solved two problems. Then add your two differences together. The player with the higher sum wins.

7 Play as many rounds as you have time for.

Blackline S 8.4 Run 1 copy, double-sided for each player.

NAME Becky DATE Oct. 10

Larger Numbers on a Line Record Sheet.

Problem Number	example	Equation Showing the Difference
		$482 - 367 = 115$
		$100 - 3 = 12 = 115$

example The baseball team was selling candy bars to raise money for uniforms. They got 482 bars and sold 367 in a week. How many did they have left over?

Larger Numbers on a Line Problem Cards

1 The milk truck delivered 457 milk cartons to the school this morning. Three hundred eighty-nine of them were plain milk, and the rest were chocolate milk. How many cartons of chocolate milk were delivered to the school?

Blackline NC 5 8.3 Larger Numbers on a Line Problem Card

2 Mrs. Olson was in charge of the book fair. She ordered 769 books and sold 583. How many books were not sold?

Blackline NC 5 8.3 Larger Numbers on a Line Problem Card

3 Kim had been collecting stickers for a long time. She counted 329 stickers in her box. She gave her new friend Maria 182 of those stickers to get her collection started. How many stickers does Kim have now?

Blackline NC 5 8.3 Larger Numbers on a Line Problem Card

4 There are 964 students at the middle school. Four hundred eighty-seven of them ride the bus to school each day, and the rest get to school some other way. How many students at the middle school do not ride the bus to school?

Blackline NC 5 8.3 Larger Numbers on a Line Problem Card

5 Kyle was going on a trip from Oregon to San Francisco, California, with his family. His dad said they would travel 672 miles in one day! By lunchtime they had driven 387 miles. How many more miles did they need to travel that day?

Blackline NC 5 8.3 Larger Numbers on a Line Problem Card

6 Five hundred twenty-eight students from our city marched in the parades this weekend. Three hundred seventy-one students marched on Saturday and the rest marched on Sunday. How many students marched on Sunday?

Blackline NC 5 8.3 Larger Numbers on a Line Problem Card

7 The local zoo is keeping track of how much their baby elephant, Panang, is growing. When Panang was 2 months old, he weighed 476 pounds. When he was 5 months old, he weighed 839 pounds. How much weight did Panang gain during those 3 months?

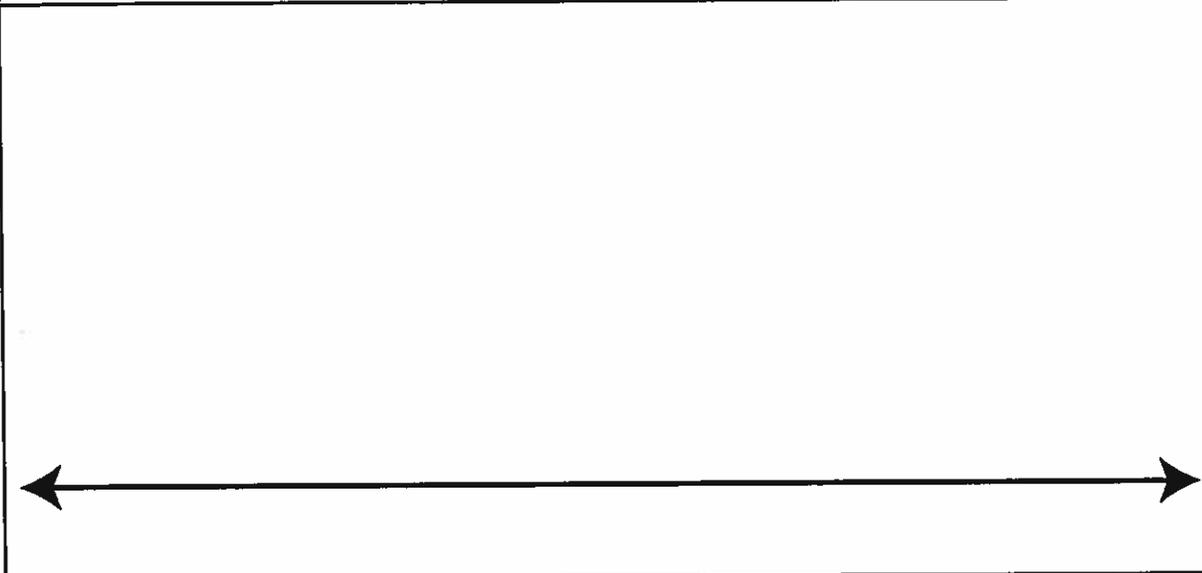
Blackline NC 5 8.3 Larger Numbers on a Line Problem Card

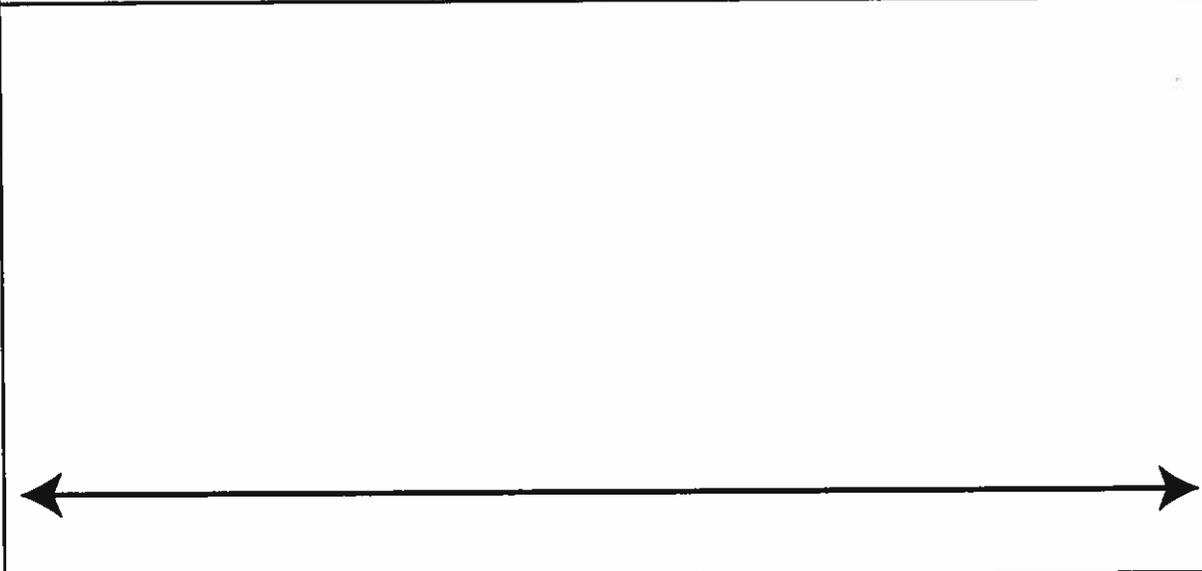
8 A mother horse weighs 922 pounds. Her foal is about a year old and weighs 657 pounds. How much more does the mother horse weigh than her foal? (A foal is a baby horse.)

Blackline NC 5 8.3 Larger Numbers on a Line Problem Card

NAME _____ DATE _____

Larger Numbers on a Line Record Sheet

Problem Number	Equations Showing the Difference
	

Problem Number	Equations Showing the Difference
	

Sum of your differences above:

Support Activity 10 ★ Instructional Considerations



SUPPORT ACTIVITY

Multiplication Challenge

Overview

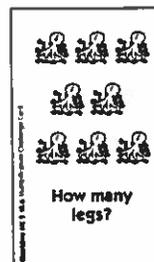
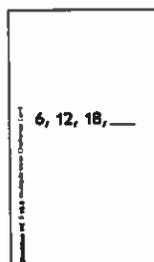
Each player draws a card that shows a multiplication fact depicted as an array, equal groups of something, or an element in a count-by sequence. Players find their products and state the complete multiplication fact. The player with the higher product wins both cards. When all cards have been played, the player with the most cards wins the game.

Skills & Concepts

- ★ using models, words, and numbers to demonstrate an understanding of multiplication
- ★ seeing multiplication as repeated addition, equal groups of objects, arrays, and skip counting
- ★ determining the relationship between two quantities
- ★ extending number patterns by adding and multiplying single-digit numbers

You'll need

- ★ Instructions for Multiplication Challenge (Blackline S 10.2)
- ★ Multiplication Challenge Cards (Blacklines S 10.3–10.5, 1 copy for each pair of players, cut apart and stored in a resealable plastic bag or envelope)



This game provides practice with skip counting, arrays, thinking algebraically about relationships between two quantities, and the grouping model for multiplication. Encourage students to name the factors and describe the strategies they are using to find the products each time they draw a new pair of cards.

Support Activity 10



SUPPORT ACTIVITY

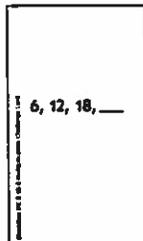
Multiplication Challenge

You'll need

- ★ Instructions for Multiplication Challenge (Blackline S 10.2)
- ★ Multiplication Challenge Cards (Blacklines S 10.3–10.5, 1 copy for each pair of players, cut apart and stored in a resealable plastic bag or envelope)

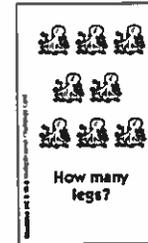
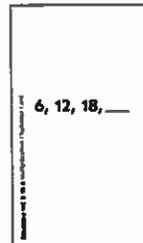
Instructions for Multiplication Challenge

- 1** Choose a partner and get a deck of cards to share.
- 2** Put the cards in a stack in front of you, face down. Decide who will go first.
- 3** The first player turns over the top card. He or she says what multiplication fact it shows.



Hmm, it's counting by 6, so it'll be 6 times 4. That's 24. I know because 5 times 4 is 20 and another 4 is 24. Also, 18 plus 6 is 24, so I'm sure it's 24.

- 4** The second player turns over a card and does the same. The player with the greatest product wins both cards. Both players must name their factors and products.



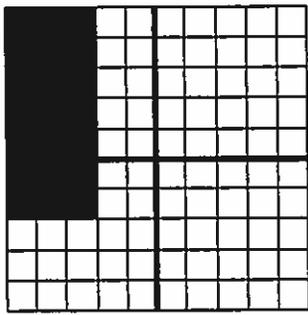
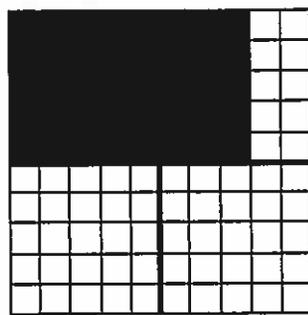
Mine is 8 times 8. That's easy for me to remember it's 64. But I could also say 8 times 4 is 32, and then double that gives you 64. I get both cards!

- 5** If both players have the same product, they each draw a second card. They name their fact and its product, and the player with the greatest product wins all 4 cards.

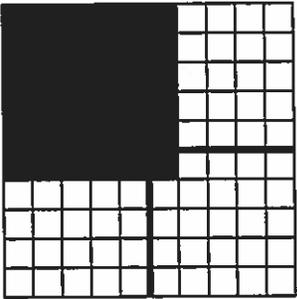
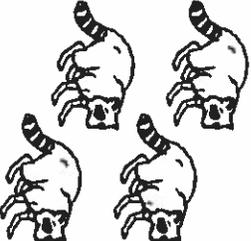
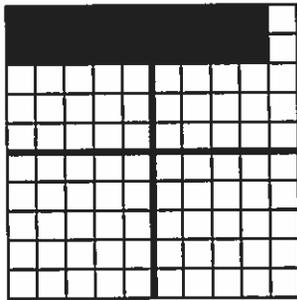
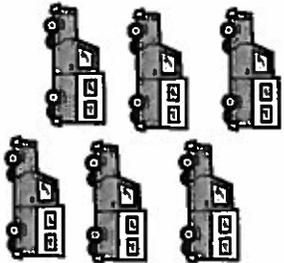
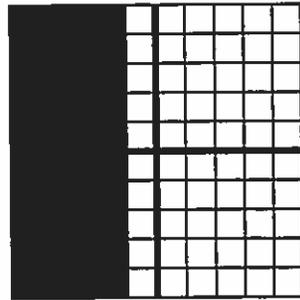
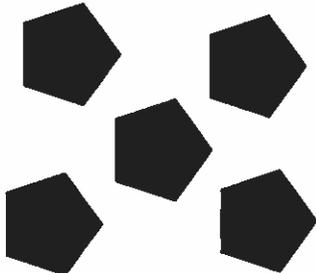
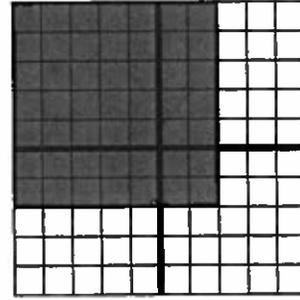
- 6** As you play, talk to each other about how you found the product for each fact. Can you think of more than one way to find the product? Do you and your partner see different ways to do it?

- 7** When there are no cards left, both players count their cards. The player with the most cards wins.

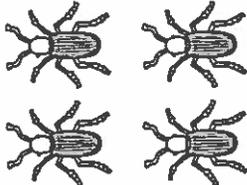
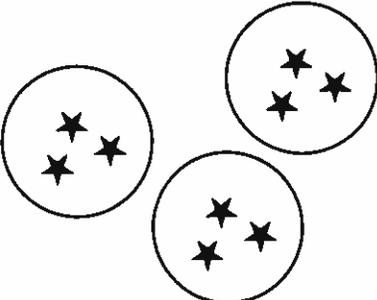
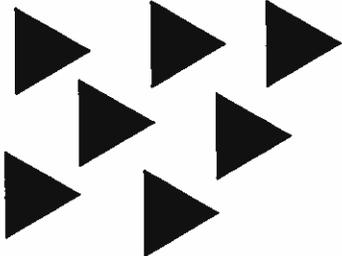
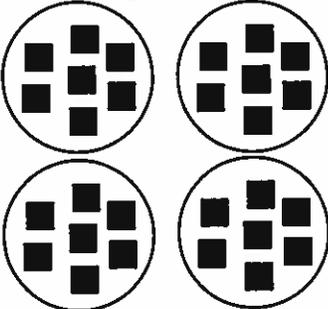
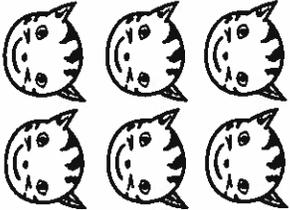
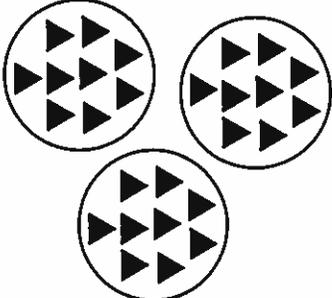
Multiplication Challenge Cards page 1 of 3

<p>Blackline NC S 14.3 Multiplication Challenge Card</p> <p style="text-align: center;">8, 16, 24, ___</p>	<p>Blackline NC S 14.3 Multiplication Challenge Card</p> <p style="text-align: center;">3, 6, 9, ___</p>
<p>Blackline NC S 14.3 Multiplication Challenge Card</p> <p style="text-align: center;">7, 14, 21, ___</p>	<p>Blackline NC S 14.3 Multiplication Challenge Card</p> <p style="text-align: center;">4, 8, 12, 16, ___</p>
<p>Blackline NC S 14.3 Multiplication Challenge Card</p> <p style="text-align: center;">What is the total area?</p> 	<p>Blackline NC S 14.3 Multiplication Challenge Card</p> <p style="text-align: center;">6, 12, 18, ___</p>
<p>Blackline NC S 14.3 Multiplication Challenge Card</p> <p style="text-align: center;">What is the total area?</p> 	<p>Blackline NC S 14.3 Multiplication Challenge Card</p> <p style="text-align: center;">9, 18, 27, ___</p>

Multiplication Challenge Cards page 2 of 3

<p>Blackline NC 5 10.4 Multiplication Challenge Card</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="border-right: 1px solid black;"></th> <th>bikes</th> <th>wheels</th> </tr> </thead> <tbody> <tr> <td style="border-right: 1px solid black;">1</td> <td></td> <td>2</td> </tr> <tr> <td style="border-right: 1px solid black;">2</td> <td></td> <td>—</td> </tr> <tr> <td style="border-right: 1px solid black;">3</td> <td></td> <td>6</td> </tr> </tbody> </table>		bikes	wheels	1		2	2		—	3		6	<p>Blackline NC 5 10.4 Multiplication Challenge Card</p> <p>What is the total area?</p> 
	bikes	wheels											
1		2											
2		—											
3		6											
<p>Blackline NC 5 10.4 Multiplication Challenge Card</p> <p>How many legs?</p> 	<p>Blackline NC 5 10.4 Multiplication Challenge Card</p> <p>What is the total area?</p> 												
<p>Blackline NC 5 10.4 Multiplication Challenge Card</p> <p>How many tires?</p> 	<p>Blackline NC 5 10.4 Multiplication Challenge Card</p> <p>What is the total area?</p> 												
<p>Blackline NC 5 10.4 Multiplication Challenge Card</p> <p>How many sides?</p> 	<p>Blackline NC 5 10.4 Multiplication Challenge Card</p> <p>What is the total area?</p> 												

Multiplication Challenge Cards page 3 of 3

<p>Blackline NC 5 10.5 Multiplication Challenge Card</p> <p>How many stars?</p> 	<p>Blackline NC 5 10.5 Multiplication Challenge Card</p> <p>How many legs?</p> 
<p>Blackline NC 5 10.5 Multiplication Challenge Card</p> <p>How many stars?</p> 	<p>Blackline NC 5 10.5 Multiplication Challenge Card</p> <p>How many angles?</p> 
<p>Blackline NC 5 10.5 Multiplication Challenge Card</p> <p>How many squares?</p> 	<p>Blackline NC 5 10.5 Multiplication Challenge Card</p> <p>How many eyes?</p> 
<p>Blackline NC 5 10.5 Multiplication Challenge Card</p> <p>How many triangles?</p> 	<p>Blackline NC 5 10.5 Multiplication Challenge Card</p> <p>How many legs?</p> 

Session 3



PROBLEMS & INVESTIGATIONS

Fractions of a Foot

Overview

Students begin the session by dividing foot-long strips in half, determining how many inches are in half a foot, and discussing their strategies for doing so. Then they divide a foot-long strip into fourths and discuss their methods before pairing off to divide more strips in as many different ways as they can. To conclude the session, they use their strips to compare fractions, explore equivalent fractions, and add fractions informally.

Actions

- 1 Students divide foot-long strips in half and determine how many inches are in half a foot.
- 2 Students divide foot-long strips in fourths and determine how many inches are in one-fourth of a foot.
- 3 Student pairs divide foot-long strips into as many different fractions as they can.
- 4 The class uses their foot-long strips to compare fractions, explore equivalent fractions, and possibly explore informal addition of fractions.

Marking Half a Foot

Begin the session by passing out a foot-long strip of construction paper to each student and making sure everyone has a ruler marked in inches. Then ask them to determine how long the strips of paper are. Most will quickly use their rulers to determine the length, but keep an eye out while students measure to see if you need to review how to use a ruler. If your classroom rulers do not begin exactly at 0, for example, you may need to review with students that they must align the end of the object they are measuring with the 0 mark.

Then ask them to make a mark exactly halfway along the strip and determine how many inches are in half a foot. Give students some time to do this,

Skills & Concepts

- ★ modeling, recognizing, and comparing common fractions
- ★ using a variety of physical and visual models to conceptualize fractions
- ★ interpreting different meanings for fractions (equal parts of a unit whole, length)

You'll need

- ★ Ruler Fractions (Overhead 3.5)
- ★ Home Connection 20, pages 65 and 66
- ★ class set of 12-inch rulers
- ★ class set plus a few more of 9" x 12" pieces of light-colored construction paper (See Advance Preparation below.)
- ★ class set of paperclips
- ★ overhead pens

.....
Advance Preparation Cut each piece of construction paper into 6 strips lengthwise, so that each measures 1.5" wide and 12" long. Use light-colored paper so that students can write on it.



Session 3 Fractions of a Foot (cont.)

and if they finish quickly, ask them to think of other ways they could identify the halfway mark. Then ask volunteers to share how they located $\frac{1}{2}$ and determined how many inches were in half a foot. Summarize their strategies on the whiteboard so students can refer to them throughout the session, and then demonstrate how the half mark should be labeled using the Ruler Fractions overhead. Ask them to label their own strips in the same way.



Ariana I remembered that 6 inches is half a foot, so I just measured and made a mark at 6 inches.

Sam I didn't remember that, but I just looked at my ruler and saw that 6 was in the middle. And 6 is half of 12, and there's 12 inches in a foot, so that made sense to me.

Nicole I just kind of looked at where I thought it was halfway.

Ricardo Did you get it right in the middle?

Nicole Here, let me measure. ... Mmm, no, not really. I put the mark at about 5 inches, not 6. I guess I should have measured first.

Raina All I did was fold it in half, and then I marked it. I didn't think about how many inches that was. But if I measure it ... yup, it's 6 inches like Ariana and Sam said.

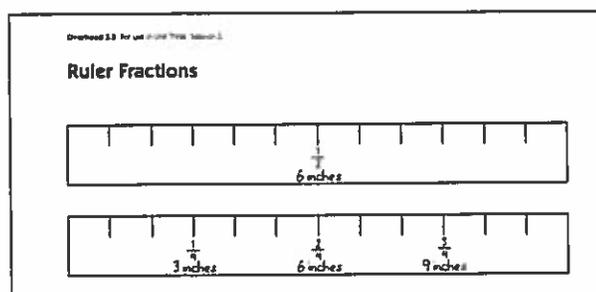
How to find half a foot:

1. Use what you know about feet – Remember 6 inches is half a foot. Then measure 6 inches and mark it.
2. Use what you know about 12 – Know that there are 12 inches in a foot. Half of 12 is 6, so measure 6 inches and mark it half a foot.
3. Fold the paper – Fold the paper foot in half. Then measure it and mark it.

Session 3 Fractions of a Foot (cont.)**Marking Fourths of a Foot**

Now give each student another strip and ask them to divide it into fourths and then label those fourths with fractions and inches using the format you just modeled on the overhead. Explain that they do not need to include markings for every inch on their strips. (These are included on the overhead to make the rulers easier to mark.) Encourage them to use any combination of strategies recorded on the board, or to think of new ways to divide the strips.

Once students have had time to work through the task, invite volunteers to label one of the rulers on the overhead and explain how they divided the strip into fourths and how they knew how to label each mark.



Be sure to have volunteers share a few different strategies. Some students may have folded the strip in half and then in half again to mark the fourths. Others may have understood that if 6 inches is half a foot, 3 inches must be a quarter of a foot because 3 is half of 6. Some may have divided 12 by 4 to arrive at 3. A few students are likely to have begun by marking off 4-inch increments. Encourage the class to talk about this mistake. Why is 4 inches not a fourth of a foot? What fraction of a foot is it, and how can they tell?

Alec I thought it was going to be 4 inches, so I made marks at 4 and 8 inches. But that doesn't seem to work.

Ricardo I folded it in half and then in half again to make fourths, and each one is 3 inches long, so 4 inches can't be a fourth.

Mayumi I think you made thirds, Alec. Because you divided it into three pieces that are all 4 inches long.

Lilly Hey, that makes sense because 12 divided by 3 is 4!

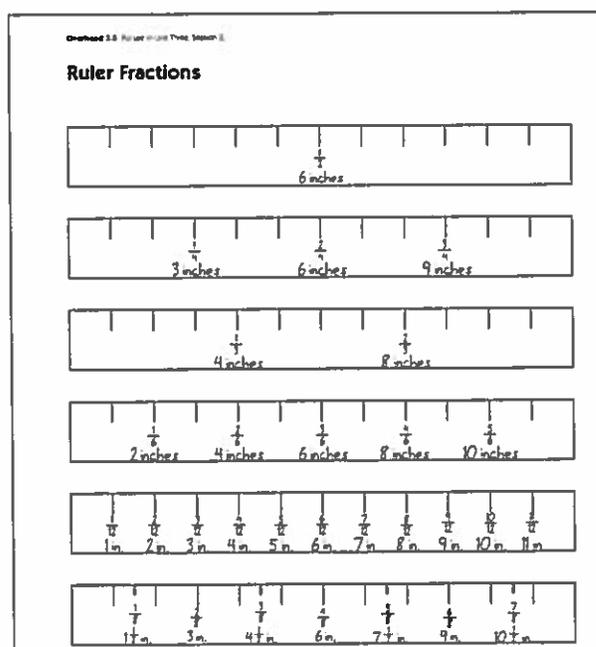
Dividing the Foot into Different Fractions

Now ask student pairs to find different ways to divide the foot-long strips into equal parts. Although they'll work together, students should divide and label their own strips so that they can re-use them in Home Connection 20. Ask them to use a new strip for each fraction and remind them to label each mark with its fraction and number of inches. Leave the strips in a central location

Session 3 Fractions of a Foot (cont.)

and ask students to come get new ones as they need them. While students work, circulate throughout the room to observe. Periodically inform the class of the progress you see, making such announcements as, "I see a pair who has found 5 different ways to divide the strip evenly," or, "At least one pair has found a way to divide the strip into eighths. Can you find a way to do that too?"

After they have had plenty of time to work, reconvene the class and have students share their fractions. Ask volunteers to label a ruler on the overhead each time a new fraction is shared, and be sure to have them explain how they made the equal divisions, keeping in mind that students are likely to have different methods for identifying and labeling the same fractions. Hand out more strips and invite students to divide them in new ways as classmates share divisions they did not create themselves.



If your students divide the strip into eighths, as shown above, they will need to make their own marks on the overhead to show half inches. If someone shares the idea of dividing the strip into eighths, be sure everyone can try it out and invite them all to measure the eighths with their rulers so that they can practice measuring to the half inch.

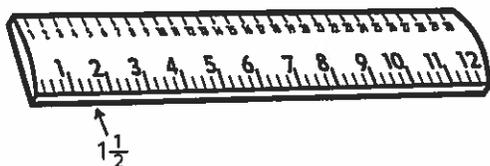
Teacher Brittany explained that she folded her paper in half three times to make eighths. Can you all please take a strip with your partner and divide it into eighths Brittany's way or some other way. Then we'll share what we noticed as a class. ...

Larry You have to use half inches on the ruler to measure them.

Session 3 Fractions of a Foot (cont.)

Sam Larry, how can you tell where the half inch is on the ruler? I don't see a half anywhere on it.

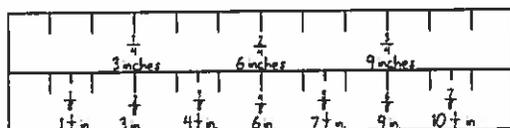
Larry The halves are always halfway between the numbers. So here's one and a half.



Becky Kathryn and I were thinking about it, and we didn't notice it before, but since 3 inches is a fourth, you can tell an eighth would be 1 and a half inches, and then you could just measure it out, one and a half inches for every eighth.

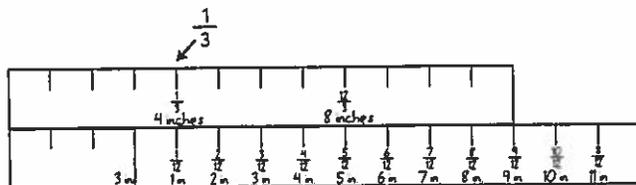
Exploring Equivalencies and Addition of Fractions

Spend the rest of the session having students explore some of problems listed below by holding up their strips side-by-side to compare fractions and demonstrate equivalencies. Ask them to record their discoveries in their journals. Leave the overhead posted so that they can refer to it while conducting their investigations.



$$\frac{2}{8} = \frac{1}{4}$$

You can pick and choose from the questions below to tailor the challenge level to meet your students' needs. Note that when adding fractions, students are likely to express the combinations as a collection of unit fractions like this $\frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$, instead of like this $\frac{1}{4} + \frac{2}{8} = \frac{1}{2}$. If you think students would enjoy the challenge of the addition problems, you might want to have them cut apart their strips first to make the fractions easier to combine. Otherwise, they can overlap their strips to combine fractions as shown below.



$$\frac{1}{4} + \frac{1}{12}$$

$$\frac{1}{4} + \frac{1}{12} = \frac{1}{3}$$

Session 3 Fractions of a Foot (cont.)

Comparing

- Which is bigger: $\frac{1}{2}$ or $\frac{1}{4}$ of a foot? ($\frac{1}{2}$)
- Which is bigger: $\frac{1}{4}$ or $\frac{1}{3}$ of a foot? ($\frac{1}{3}$)

Finding Equivalencies

- What other fractions are equal to half of a foot? ($\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{6}{12}$)
- What other fractions are equal to one-third of a foot? ($\frac{2}{6}, \frac{4}{12}$)
- What other fractions are equal to two-thirds of a foot? ($\frac{4}{6}, \frac{8}{12}$)
- What other fractions are equal to one-fourth of a foot? ($\frac{2}{8}, \frac{3}{12}$)
- What other fractions are equal to three-fourths of a foot? ($\frac{6}{8}, \frac{9}{12}$)

Adding

- What fraction do you get when you add $\frac{1}{2} + \frac{1}{4}$? ($\frac{3}{4}$)
- What fractions can you add together to equal half a foot? ($\frac{1}{4} + \frac{1}{4}, \frac{1}{4} + \frac{1}{8} + \frac{1}{8}, \frac{1}{6} + \frac{1}{6} + \frac{1}{12} + \frac{1}{12}$, etc.)
- What fractions can you add together to equal one-third of a foot? ($\frac{1}{6} + \frac{1}{6}, \frac{1}{6} + \frac{1}{12} + \frac{1}{12}, \frac{1}{4} + \frac{1}{12}$, etc.)



HOME CONNECTION 20

In Home Connection 20 students use their foot-long strips to compare and add fractions of a foot. Before sending the assignment home, give each student a paperclip so they can affix their strips to their papers.

Home Connections For use after Unit Three, Session 3
 NAME _____ DATE _____

Home Connection 20 ★ Worksheet

Comparing Fractions of a Foot
 Use your labeled foot-long strips to answer the questions below:

- 1 Which is larger: $\frac{1}{2}$ of a foot or $\frac{1}{4}$ of a foot? _____
- 2 Which is larger: $\frac{1}{3}$ of a foot or $\frac{1}{4}$ of a foot? _____
- 3 Which is larger: $\frac{1}{2}$ of a foot or $\frac{2}{3}$ of a foot? _____
- 4 Which is larger: $\frac{2}{3}$ of a foot or $\frac{3}{4}$ of a foot? _____
- 5 What do you get when you add $\frac{1}{2}$ of a foot and $\frac{1}{4}$ of a foot? _____

Quick Facts Practice

6 Multiply the number in each small box below by the number shown.

x	5	7	3	9	6	4	2	8
3	15	21	9	27	18	12	6	24

x	5	7	3	9	6	4	2	8
4								

x	5	7	3	9	6	4	2	8
6								

x	5	7	3	9	6	4	2	8
8								

Home Connections
 Home Connection 20 Worksheet (cont.)

CHALLENGE

- 7 What do you get when you add $\frac{1}{2}$ of a foot and $\frac{1}{3}$ of a foot? _____
- 8 How many different combinations of fractions can you add together to equal $\frac{1}{2}$? Write them below or on another piece of paper.

Session 4



PROBLEMS & INVESTIGATIONS

Introduction to Egg Carton Fractions

Overview

Students generate a list of things that come in groups of 12 and are then introduced to the 12-egg carton as a model for representing fractions as parts of a whole and parts of a set. Then students use the egg carton to model different unit fractions (e.g., $\frac{1}{2}$, $\frac{1}{3}$, etc.) and review the terms *numerator* and *denominator*.

Actions

- 1 Students write about fractions in their journals.
- 2 The class generates a list of things that come in groups of 12.
- 3 The teacher introduces the egg carton as a model for fractions.
- 4 Students build models of unit fractions.

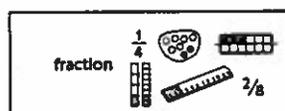
Skills & Concepts

- ★ using physical and visual models to conceptualize fractions
- ★ interpreting fractions as equal parts of a unit whole or equal parts of a set

Journaling about Fractions

Ask students to spend a few minutes writing anything they know or have experienced about fractions in their journals. Encourage them to use both words and labeled sketches. You might write prompts like the following on your whiteboard and post the *fraction* Word Resource Card to help get students started:

- Give some examples of fractions. Use numbers and/or drawings.
- What does a fraction mean?
- What models have you used to represent fractions in other grades?



You'll need

- ★ Egg Carton Fractions (Overhead 3.6)
- ★ Word Resource Cards (fraction, denominator, numerator)
- ★ 12 tile per student
- ★ class set of egg cartons
- ★ six 10-inch lengths of heavy string or yarn per student
- ★ 24" by 36" piece of butcher paper (see Advance Preparation below)
- ★ overhead pens
- ★ wide-line felt-tip pen

.....
Advance Preparation Tape a 24" by 36" sheet of butcher paper to the wall or whiteboard in an easily accessible spot.



Session 4 Introduction to Egg Carton Fractions (cont.)

You might look over students' responses later to get a sense of what they know about fractions, and have them add to this page as they learn more in the weeks to come.

.....
Best Practice Tip Asking students to recall and communicate what they already know about a topic increases student achievement by allowing them to make vital connections between previous knowledge and new learning. These connections support the retention of new information and skills.

Creating a List of Things That Come in Groups of 12

Now ask students to pair up and brainstorm a list of things that come in groups of 12 and write them in their journals. Give them time to come up with several ideas and then ask volunteers to share some of their ideas while you list them on the large sheet of butcher paper. Invite students to add new ideas to the lists in their journals.

Things that come in 12's

- a dozen donuts
- 12 inches in a foot
- 12 candles on my sister's birthday cake
- 12 eggs in a carton
- 12 hours on the classroom clock
- 12 new pencils in a package
- Sam earned \$12 washing cars last weekend.
- There are 12 of anything in a dozen.
- 12 golf balls in a box
- 12 squares in a giant chocolate bar
- Raina's grandpa's boat is 12 feet long.
- 12 months in a year.

Leave this list posted for the next few days so that students can add items to it as they think of them.

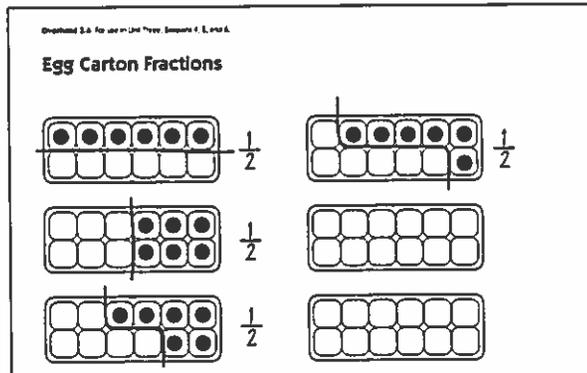
Introducing the Egg Carton as a Model for Fractions

If students don't mention eggs as something that comes in 12's, add eggs to the class list and then give each student 1 egg carton, 12 tile, and six 10-inch lengths of heavy string or yarn. Ask students to set aside their yarn for a minute and use the tile to fill their egg cartons completely, sharing with their tablemates any observations they can make about how the eggs are arranged. Then invite volunteers to share observations with the class.

Write $\frac{1}{2}$ on the whiteboard and ask students to divide the carton in half with the string and then fill in one-half with tile. Once they have created models, ask students to pair up and explain how their models show the meaning of one-half.

Session 4 Introduction to Egg Carton Fractions (cont.)

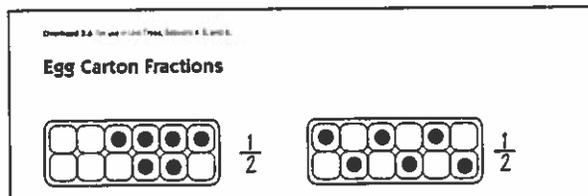
Now ask for volunteers to sketch their models on the Egg Carton Fractions transparency as they explain their thinking to the class. Encourage them to make a dot for each tile and draw a line to show where they placed the string or yarn, and have each volunteer label his or her sketch $\frac{1}{2}$. Ask students to make observations about how the various models are similar and how they are different.



David Some of the ways people showed look different at first, but when you count up the little boxes in each part, each one always has 6.

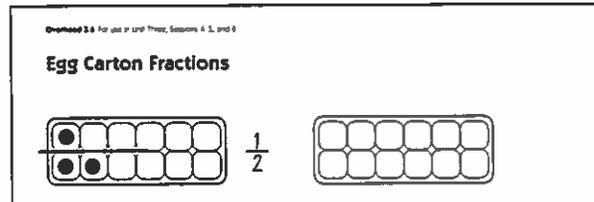
Raina This is like what we did the other day when we made halves on the geoboard that had lots of different shapes.

Some students may suggest dividing the carton so that the two halves are not represented by contiguous arrangements of compartments, as shown below. If so, affirm that their divisions do represent half of the carton, but that for now, you'd like them to stick with models that keep all the "eggs" together in a group. This makes it easier for students to see each model as part of a whole and part of a set.



One or more students might create a literal depiction of the symbolic representation, as shown on the next page. If so, acknowledge students' thinking without making corrections. Subsequent activities and interactions with classmates over the next few sessions will provide opportunities for students to adjust their conceptions of fractions.

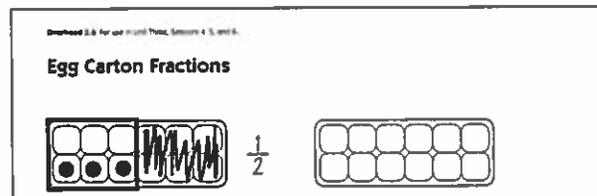
Session 4 Introduction to Egg Carton Fractions (cont.)



student's literal depiction of $\frac{1}{2}$

A student might change the size of the whole by creating a carton of a different size. If so, use it as an opportunity to discuss the relevance of the size of the whole. If not, you might propose it to spark this discussion. In our classrooms, we generally solicit agreement from the class to keep the whole the same size throughout the investigation for consistency.

Teacher *What if I changed it into a 6-egg carton and divided it like this? Does this show one-half? Please talk to your partner about whether it shows one-half and whether you think this is a good way to model half with the 12-egg carton. ...*



Sam *We said it shows a half. You kind of forget about the 6 that are crossed out and then 3 is half of 6. It works.*

Brittany *It works, but I think we should just stick with using the whole thing. If we have the whole carton, I think we should use it.*

Use this conversation to segue into a conversation about the whole. Write *one whole* on the whiteboard or overhead and ask students to show that amount in their egg cartons and discuss their results with their tablemates.

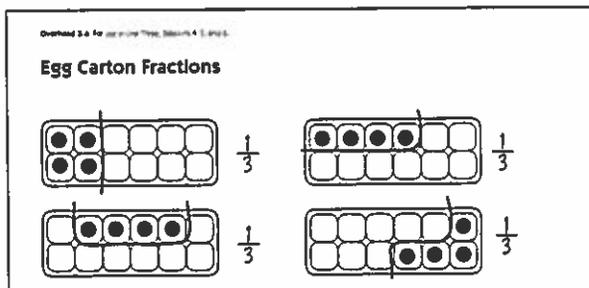
- What does one whole look like with the 12-egg carton model? Is it one egg? Is it the whole carton full of eggs?
- How can *one* possibly mean 12?
- Can students think of other situations where *one* means 12?

If the word *dozen* hasn't yet come up, introduce it now by explaining that the carton holds 12 separate eggs, which together make 1 dozen. The whole for the egg carton fraction model can be viewed as the 2×6 array formed by the carton, one whole dozen, or one set of 12 eggs. Most visual or physical models for fractions specifically portray either the parts-of-a-whole or parts-of-a-set concept of fractions. The egg carton is a powerful fraction model, because it simultaneously models both concepts.

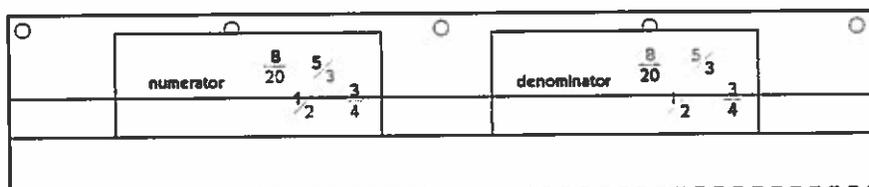
Session 4 Introduction to Egg Carton Fractions (cont.)

Building Unit Fractions

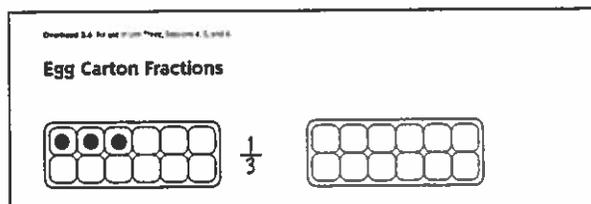
Write $\frac{1}{3}$ on the whiteboard and ask students to use their egg cartons to show one-third. Give students time to discuss their models with partners and then ask a few volunteers to come to the overhead to sketch, label, and explain the thinking behind their models. Again, ask students to look for similarities between the different models.



There will probably be a few students who maintain that 3 eggs show $\frac{1}{3}$. If so, ask students to explore this proposition, although you'll probably find that students continue to struggle with these kinds of misconceptions for days to come. During this discussion, be sure to review the meaning of the words *numerator* and *denominator* using the Word Resource Cards as necessary. Be sure students explore what the numbers in each position in the fraction mean, and record their ideas on the board.



David Well, it says one, three, so I just made 1 group of 3, like this.



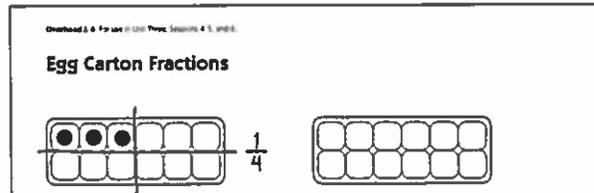
Ariana I don't think that works, David, because the other ones all have 4 eggs in them.

Teacher What does everyone else think? Please talk to your partners about this for a minute, and then we'll get back together to share some ideas. ...

Session 4 Introduction to Egg Carton Fractions (cont.)

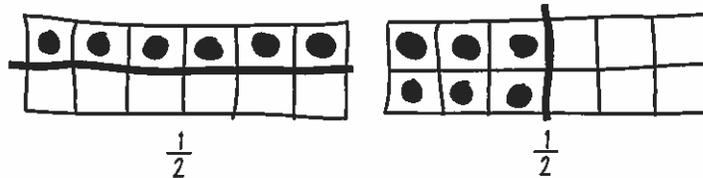
Antoine We think it has to be 4 eggs, because a third of 12 is 4. Because 12 divided by 3 is 4.

Lilly We thought David's idea might work, so we kept making groups of 3 with our yarn like this. But then there were 4 equal pieces, so we thought that 3 would show a fourth, not a third.



Keith But if it's 3, I don't get how it can be a fourth!

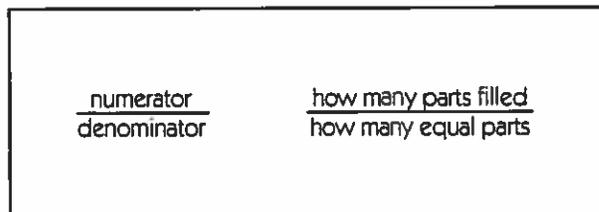
Teacher It sounds like we need to take a minute to think about what these numbers in the fraction mean. In all fractions, we call the top number the numerator and the bottom number the denominator. Let's think about the example of one-half for a minute. What does the numerator, 1, tell you, and what does the denominator, 2, tell you? Or, where do you see the 1 and the 2 in the models we drew? I'll draw a few on the board to refresh your memory.



Cheyenne Well, the 2 tells you it's a half, that there are 2 parts.

Sam And that they're the same size.

Alena The 1 just tells you there's 1 of them, just one half, not two, filled in.



Teacher So, continuing with those ideas, what do the numbers in one-third tell you?

Raina 3 would be how many equal parts and 1 is how many are filled in!

Session 4 Introduction to Egg Carton Fractions (cont.)

Now ask students to work in pairs to build as many other unit fractions as they can. Explain that a unit fraction is any fraction with a 1 in the numerator (read "one-something," as in one-half, one-third, or one-fourth). Two-thirds, for example, would not be a unit fraction. As they work, have them record quick labeled sketches of the fractions in their journals. As students work, talk with individuals and groups about their thinking. Periodically tell the class how many unit fractions others have found to spur them to keep thinking and experimenting (e.g., Raina has found 2 new unit fractions). They will be able to create 6 unit fractions ($\frac{1}{12}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{1}$), although each fraction, with the exception of $\frac{1}{1}$, can be modeled in a variety of ways. You might encourage partners to model each one differently. If your students don't mention $\frac{1}{1}$, you might propose it yourself or let the issue pass.



Student Name: Rafael DATE: Nov. 30
 NAME: _____
 TITLE: Unit Fractions

	$\frac{1}{4}$	<input type="text"/>
	$\frac{1}{3}$	<input type="text"/>
	$\frac{1}{6}$	<input type="text"/>

Student Name: Nila DATE: Nov. 30
 NAME: _____
 TITLE: Unit Fractions

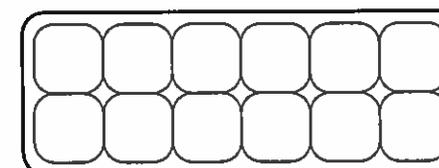
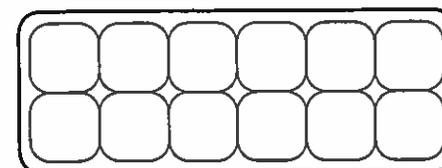
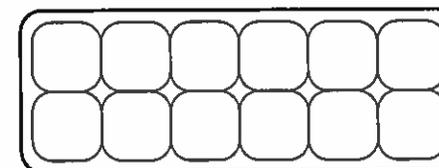
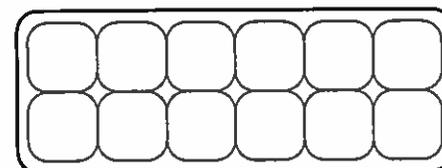
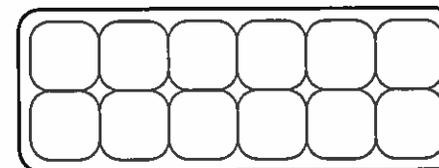
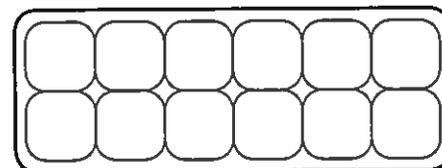
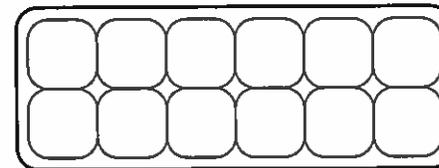
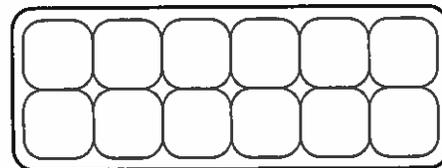
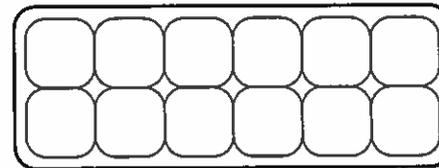
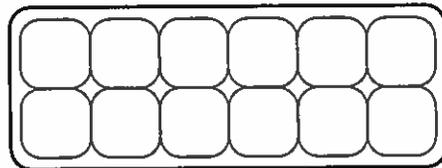
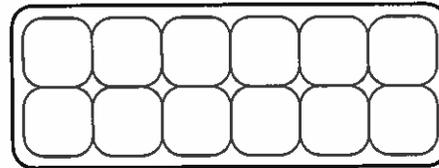
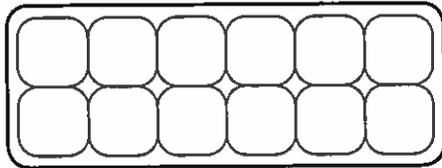
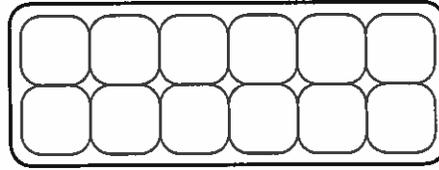
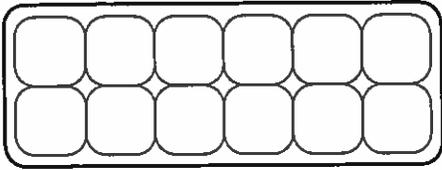
	$\frac{1}{4}$		$\frac{1}{6}$
	$\frac{1}{3}$		

Blackline 3.1 For use in Unit Three, Sessions 5, 6, and 8. Run a double class set plus a few extra. Don't run double sided.

NAME _____

DATE _____

Egg Carton Fractions Record Sheet



Session 9



PROBLEMS & INVESTIGATIONS

Fractions on a Clock

Overview

Students use an analog clock face as another model for fractions. The concepts and activities in this session serve to increase students' comfort with the clock face, as well as with fractions. The session ends with students using this new model to create a diagram of how a student spent his time outside after school. The teacher assigns Home Connection 23, Clock Fractions, today.

Actions

- 1 Students model fractions on clock faces.
- 2 Students make a journal record of their findings.
- 3 Students use a clock face to model how a student spent his time outside after school.
- 4 The teacher assigns Home Connection 23.

Skills & Concepts

- ★ using a variety of physical and visual models to conceptualize fractions
- ★ interpreting different meanings for fractions (equal parts of a unit whole, time)
- ★ using visual models to add common fractions

You'll need

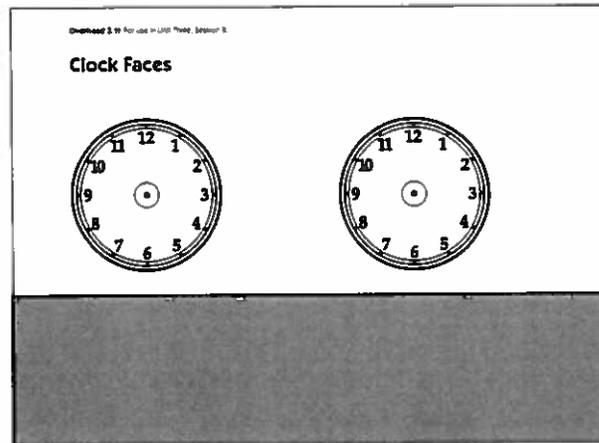
- ★ Clock Faces (Overhead 3.11)
- ★ After School (Overhead 3.12)
- ★ Small Clock Faces (Bridges Student Book, page 50)
- ★ Complete Clock Faces (Bridges Student Book, page 51)
- ★ After School (Bridges Student Book, page 53)
- ★ Home Connection 23, pages 71 and 72
- ★ half-class set of student clocks
- ★ class set of rulers
- ★ scissors
- ★ glue
- ★ card or sheet of paper for masking portions of the transparencies



Modeling Fractions on Clock Faces

Show the top two clock faces on the Clock Faces transparency while masking the rest of the sheet. Explain that the hands were left off so that students can focus on the other attributes of the clock face.

Session 9 Fractions on a Clock (cont.)



Ask students to study the clocks on the overhead and write observations about them in their journals. Have them describe what they see and consider the following two questions. What connections can they see between the clock faces and something they have studied already in math? What mathematical questions that don't involve time can you answer using a clock face? After two or three minutes, ask them to share their ideas first in pairs and then as a whole class.

Students They have numbers 1 to 12 on them.

You can use a clock to count by 5's or multiply by 5's.

It reminds me of circle fractions, like those pizzas and graphs last year.

Next, ask students to pair up and think about fractions on a clock face. Have they heard people speak of time in fractions? How would those fractions relate to a clock face? If a clock face is used to model fractions, what is the whole? Give partners time to think, talk, and share, and then resume the whole group discussion.

Students may think of the whole in several ways: the clock face itself, 12 hours, 1 hour or 60 minutes, or 12 five-minute chunks. Most students will be familiar with half an hour and a quarter of an hour. Clarify that a *quarter* is another way of saying *one-fourth* if you think some of your students may be confused by these synonyms. When people speak of half an hour and a quarter of an hour, they are considering one hour to be the whole, and the clock face can work very effectively as a model for fractions of an hour.

Have students turn to Small Clock Faces on page 50 in their student books, and unmask the rest of the overhead. Ask students to use their rulers and pencils to draw lines dividing one of their clock faces into halves and another into quarters and then label each segment. Ask two volunteers to do the work on the overhead. Then ask students to talk about the ways in which they can think about $\frac{1}{2}$ and $\frac{1}{4}$ on a clock.

.....
Best Practice Tip Be sure to clarify new terms and synonyms for English-language learners.

Session 9 Fractions on a Clock (cont.)

Bridges Student Book For use in Unit Three, Session 9
 NAME _____ DATE _____

Small Clock Faces

The image shows four small clock faces. The first is divided into two equal halves, with the fraction $\frac{1}{2}$ written in each half. The second is divided into four equal quadrants, with the fraction $\frac{1}{4}$ written in each quadrant. The third is divided into three equal sectors, with the fraction $\frac{1}{3}$ written in each sector. The fourth is a standard clock face with no internal divisions.

Students 6 is half of 12.
 30 is half of 60.
 30 minutes is half an hour.
 12 is half of 24. There are 24 hours in a day and half of them fit on this kind of clock.

Teacher How might you think about fourths on a clock?

Students There are 4 fourths in an hour.
 Each quarter of an hour is 15 minutes long.
 $\frac{2}{4}$ of an hour is the same as $\frac{1}{2}$ an hour.
 They're the same! $\frac{2}{4}$ and $\frac{1}{2}$ are both 30 minutes long.
 $\frac{3}{4}$ of an hour is exactly between $\frac{1}{2}$ an hour and a whole hour.
 $\frac{3}{4}$ of an hour is 45 minutes.

Finding More Clock Fractions

Now ask students to find other fractions of an hour that can be represented on a clock face. How many other ways can a clock face be divided into pieces of equal size? Invite them to experiment using the other clock faces on their sheet and the student clocks if they find them helpful. Encourage them to use a ruler when drawing their fractions to assure equal sized pieces and to make it easier for others to interpret their work. Encourage students who might enjoy the challenge to use the Complete Clock Faces on page 51. With all the markings shown, some students might divide the clock into equal parts that are factors of 60, such as tenths, fifths, and fifteenths. You might pair up students who are both working on the complete clocks so that they can share ideas as they work.



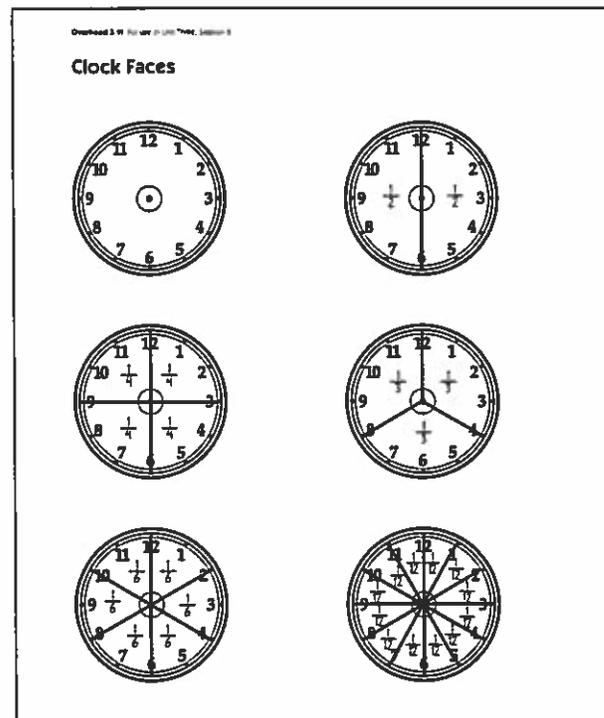
Bridges Student Book For use in Unit Three, Session 9
 NAME _____ DATE _____

Complete Clock Faces

The image shows three complete clock faces. The first is divided into 12 equal sectors, with the fraction $\frac{1}{12}$ written in each sector. The second is divided into 6 equal sectors, with the fraction $\frac{1}{6}$ written in each sector. The third is divided into 30 equal sectors, with the fraction $\frac{1}{30}$ written in each sector.

Session 9 Fractions on a Clock (cont.)

After they have had a chance to think, work, and share their ideas in pairs, invite several volunteers to each draw one of their fraction ideas on the overhead. The example below shows the ideas students are most likely to come up with in addition to halves and fourths: thirds, sixths, and twelfths. Remain open to other possibilities as well.



As the discussion proceeds, some students may begin to see connections between clock fractions and egg carton fractions. If they don't mention it themselves, you might ask if they see any similarities between the two models.

Teacher *How would you compare the clock model for fractions to the egg cartons?*

Students *In the egg carton there can be 12 eggs, and on the clock, there are 12 numbers.*

You can see lots of the same fractions on a clock as you can with an egg carton—halves, fourths, thirds, sixths, twelfths—because there are 12 places for eggs and 12 numbers on the clock.

There are 12 numbers on a clock but it's strange, because if you think about it, there's not really 12 of anything in an hour.

An hour has 60 minutes, not 12.

The 12 numbers are for the hours. The clock shows 12 hours.

When you count 5, 10, 15, 20 as you go around the clock you're counting minutes.

Session 9 Fractions on a Clock (cont.)

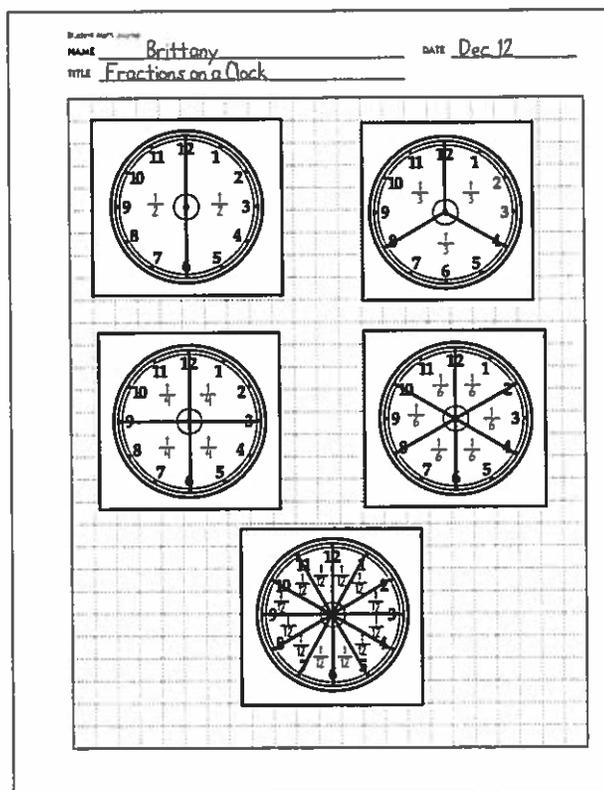
You count by fives at the hour numbers. Hey, I know! The 12 things are chunks of minutes—five minutes in each chunk!

Making a Journal Record

Now have students create a record of their clock fractions that they can refer to in future sessions. Have them title a fresh journal page “Fractions on a Clock,” on which they can glue the clock fractions they created. Before they begin, ask students to suggest a few efficient ways to cut out the little clock faces. For instance, cutting a quick square around the clock is a lot more efficient than cutting exactly along the circumference of the circle.



Suggest that, before gluing the clock fractions onto the journal page, students consider different arrangements and choose the one that makes the most mathematical sense to them. The arrangement below, for example, goes in order of increasing number of pieces. Students will also think of other ways to organize their clocks.



Thinking about Fractions of an Hour

To conclude the session, have students turn to After School on page 53 in their student books. Display the After School overhead as well, read it out loud, and give students some time to consider the situation quietly. Invite any observations or questions they might have about the task at hand and then set them

Session 9 Fractions on a Clock (cont.)

to work. If students are confused about how to record the times on the clock, ask them how they would show 5 minutes on the clock face. Then use their answers as a springboard to the larger problem of how to show 20 and 25 minutes.

Then have students diagram the situation on the clock face at the top of the sheet and record some thoughts and mathematical observations about the situation at the bottom of the sheet.

Notice that both students below can identify 20 minutes as $\frac{1}{3}$ an hour and 15 minutes as $\frac{1}{4}$ an hour, but that neither is initially able to connect 25 minutes to any fraction they know. Chris, in fact, reports that the time spent biking, "can't have a fraction because it's not even."

Bridges Student Book For Use in Unit Three, Session 9
 NAME Chris DATE _____

After School

After school yesterday John spent 20 minutes playing fetch with his dog and 25 minutes riding his bike.

1 Use the clock to draw a diagram showing how John spent his time outside.

2 In the space below, use words, numbers, and/or sketches to describe any mathematical observations you can make about this situation. Use the 2 clock faces below if you like.

I know fetch beacause when you put the clock like this. Only use one.

bike can't have a fraction because it is not even.

Bridges Student Book For Use in Unit Three, Session 9
 NAME Heidi DATE _____

After School

After school yesterday John spent 20 minutes playing fetch with his dog and 25 minutes riding his bike.

1 Use the clock to draw a diagram showing how John spent his time outside.

2 In the space below, use words, numbers, and/or sketches to describe any mathematical observations you can make about this situation. Use the 2 clock faces below if you like.

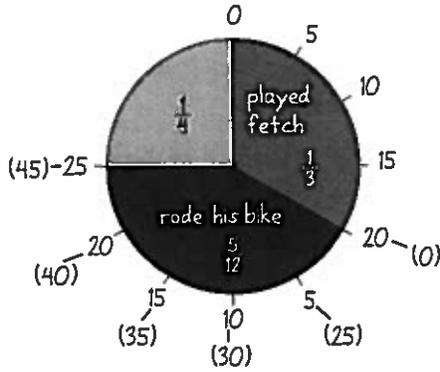
$\frac{1}{4} + 25 < 1$ hour

I know that John did nothing for $\frac{1}{4}$ of an hour. He played fetch for $\frac{1}{3}$ of an hour. He rode his bike for 25 min. Altogether he spent 45 min. outside. He spent the most time riding his bike. He spent the least time doing nothing.

After they've had time to work individually, ask students to share their ideas with partners, adding to their own sheets as they trade ideas and generate more observations together. Finally, invite students to share their ideas with the class, and record their thoughts on the overhead or whiteboard. You might inspire discussion by asking students to compare the two times in the diagram. The chart on the next page shows examples of observations our fourth graders have made. Your class will surely generate others.

Session 9 Fractions on a Clock (cont.)

Math Relationships We See in Our Diagrams



$20 + 25 = 45$ minutes John played outside
 $\frac{1}{3}$ hour playing fetch
 $\frac{5}{12}$ hour biking
 $\frac{1}{3} + \frac{5}{12} < 1$

$\frac{1}{4}$ hour more biking (or playing fetch) and he would have been outside a whole hour.

$\frac{5}{12}$ of an hour is 25 minutes.
 5 groups of 5 minutes: $5 \times 5 = 25$

John biked longer than he played fetch...
 but only 5 minutes longer.
 $\frac{5}{12}$ is just a little more than $\frac{1}{3}$.

$\frac{1}{3}$ of an hour is 20 minutes
 4 groups of 5 minutes $4 \times 5 = 20$

$\frac{1}{3} + \frac{5}{12} + \frac{1}{4} = 1$ $\frac{1}{3} + \frac{5}{12} = \frac{3}{4}$ $\frac{1}{4} + \frac{3}{4} = 1$



HOME CONNECTION 23

Students use clock faces to solve a story problem and to complete a table converting fractions of an hour to minutes in Home Connection 23, Clock Fractions.

Home Connection For use after Unit Three, Session 9

NAME _____ DATE _____

Home Connection 23 ★ Worksheet

Clock Fractions

1 Steve started reading his chapter book at 6:30 and stopped at 7:15.

a How many minutes did he read? _____

b What fraction of an hour is that? _____

c Is that closer to half an hour or an hour?

2 Complete the table below. Use the clock faces if they will help you.

fraction of an hour	minutes
$\frac{1}{2}$	
$\frac{2}{3}$	
$\frac{3}{4}$	
$\frac{1}{3}$	
$\frac{2}{4}$	
$\frac{3}{4}$	
$\frac{1}{4}$	

Fact Fluency with 5's Multiplying & Dividing by 5



MULTIPLICATION FACT FLUENCY

Strategy	How It Works	Example
Half-Decade	To multiply any number by 5, multiply it by 10 first and then divide the answer in half.	What is 5×8 ? It's 10×8 divided in half. $5 \times 8 = (10 \times 8) \div 2$ $= 80 \div 2 = 40$

1 Multiply each number in the grid by 5. Write each product in the box. The first one is done for you.

25 ⁵	7	3	9	11	8	12	6	2
10	8	11	1	9	5	0	12	4

2 Use the half-decade strategy to help solve these combinations.

$5 \times 15 = \underline{\quad}$
 $5 \times 20 = \underline{\quad}$
 $5 \times 25 = \underline{\quad}$
 $5 \times 45 = \underline{\quad}$

$$\begin{array}{r} 30 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 80 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ \times 5 \\ \hline \end{array}$$

3 Use what you know about multiplying by 5 to solve these division problems.

$30 \div 5 = \underline{\quad}$
 $45 \div 5 = \underline{\quad}$
 $55 \div 5 = \underline{\quad}$
 $35 \div 5 = \underline{\quad}$

$$5 \overline{)50}$$

$$5 \overline{)40}$$

$$5 \overline{)25}$$

$$5 \overline{)60}$$

NAME _____

DATE _____

Fact Fluency with 5's Practice Multiplying by 5 & 10



MULTIPLICATION FACT FLUENCY

1 Circle all the half-decade facts ($\times 5$) in blue. Then go back and fill in the answers with regular pencil.

2 Circle all the decade facts ($\times 10$) in red. Then go back and fill in the answers with regular pencil.

$\begin{array}{r} 1 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ \times 10 \\ \hline \end{array}$	$\begin{array}{r} 10 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 10 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ \times 10 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ \times 10 \\ \hline \end{array}$
--	--	--	---	---	---	--

$\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 10 \\ \hline \end{array}$	$\begin{array}{r} 20 \\ \times 5 \\ \hline \end{array}$
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$\begin{array}{r} 4 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 23 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 25 \\ \times 10 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ \times 10 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 10 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ \times 5 \\ \hline \end{array}$
--	--	---	--	--	---	---

3 Write two multiplication and two division facts for each set of numbers.

<p>a</p> <p>_____ \times _____ = _____</p> <p>_____ \times _____ = _____</p> <p>_____ \div _____ = _____</p> <p>_____ \div _____ = _____</p>	<p>b</p> <p>_____ \times _____ = _____</p> <p>_____ \times _____ = _____</p> <p>_____ \div _____ = _____</p> <p>_____ \div _____ = _____</p>	<p>c</p> <p>_____ \times _____ = _____</p> <p>_____ \times _____ = _____</p> <p>_____ \div _____ = _____</p> <p>_____ \div _____ = _____</p>	<p>d</p> <p>_____ \times _____ = _____</p> <p>_____ \times _____ = _____</p> <p>_____ \div _____ = _____</p> <p>_____ \div _____ = _____</p>
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NAME _____

DATE _____

Fact Fluency with 5's Division Capture 5's & 10's



MULTIPLICATION FACT FLUENCY

You'll need

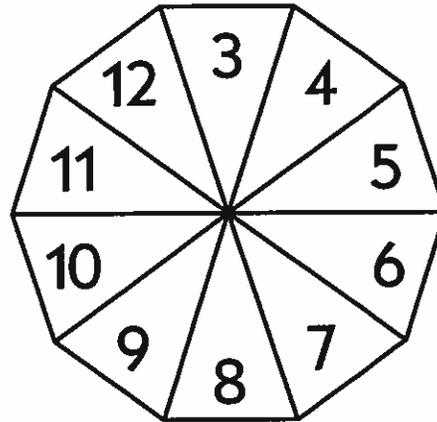
- ★ a partner
- ★ 2 pencils or markers in different colors
- ★ paperclip and pencil to use as a spinner

Instructions for Division Capture 5's & 10's

- 1** Take turns spinning the spinner. The player who gets the higher number goes first.
- 2** Take turns spinning the spinner. Use the number you spin to fill in the answer to one of the division problems below. Be sure to use your own color pencil.
- 3** If the box you need is already filled, you lose your turn.

4 Try to capture 3 or 4 boxes in a row: across, up and down, or diagonally. Keep playing until the gameboard is filled or neither player can use the number he or she spins 3 times in a row.

5 Then circle the places on the grid where you got 3 or 4 in a row and add up your scores.



$15 \div 5$	$50 \div 5$	$60 \div 5$	$60 \div 10$	$35 \div 5$	Scoring 3 in a Row—1 point 4 in a Row—2 points	
$80 \div 10$	$20 \div 5$	$30 \div 10$	$55 \div 5$	$90 \div 10$		
					Player 1 Points	Player 2 Points
$120 \div 10$	$40 \div 5$	$100 \div 10$	$30 \div 5$	$40 \div 10$		
$70 \div 10$	$25 \div 5$	$45 \div 5$	$50 \div 5$	$110 \div 10$		

NAME _____

DATE _____

Fact Fluency with 5's Flashcard Bingo 5's



MULTIPLICATION FACT FLUENCY

You'll need

- ★ a partner
- ★ one set of 5's flashcards and your flash-card pocket
- ★ marker or crayon for each player

Instructions for Flashcard Bingo 5's

1 Mix up one set of flashcards and arrange them so that the multiplication side is facing up on all of them. Put them in the flashcard pocket.

2 Take turns pulling 1 card. Each time, both players color the product on their board or mark it with an x.

3 The first player to get two rows of 4 going horizontally, vertically, or diagonally, wins.

4 Play the game a second time using the division side of your cards.

		Player 1			
Game 1 x		5	25	20	10
		60	10	25	55
		50	5	15	45
		30	35	20	40

		Player 2			
		30	5	35	50
		35	20	15	40
		45	30	60	20
		40	55	10	45

		Player 1			
Game 2 ÷		3	6	7	4
		8	11	12	6
		5	9	10	7
		1	8	5	2

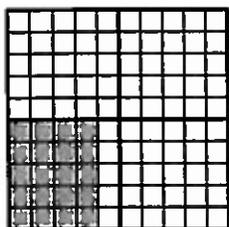
		Player 2			
		1	6	11	10
		5	10	9	2
		12	4	8	12
		9	11	3	7

Fact Fluency with 5's Flashcards, page 1 of 2

- Cut out this set of 12 flashcards.
- Fold a 3-by-5 index card in half.
- Tape or staple both sides but leave the top open.
- Label this storage pocket with your name and the set number.

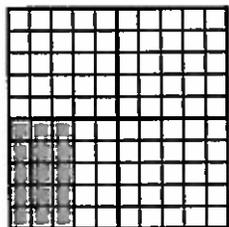
$\begin{array}{r} 1 \\ \times 5 \\ \hline \end{array}$ <p>Blackline F 23</p>	$\begin{array}{r} 2 \\ \times 5 \\ \hline \end{array}$ <p>Blackline F 23</p>	$\begin{array}{r} 3 \\ \times 5 \\ \hline \end{array}$ <p>Blackline F 23</p>	$\begin{array}{r} 4 \\ \times 5 \\ \hline \end{array}$ <p>Blackline F 23</p>
$\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$ <p>Blackline F 23</p>	$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$ <p>Blackline F 23</p>	$\begin{array}{r} 7 \\ \times 5 \\ \hline \end{array}$ <p>Blackline F 23</p>	$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$ <p>Blackline F 23</p>
$\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}$ <p>Blackline F 23</p>	$\begin{array}{r} 10 \\ \times 5 \\ \hline \end{array}$ <p>Blackline F 23</p>	$\begin{array}{r} 11 \\ \times 5 \\ \hline \end{array}$ <p>Blackline F 23</p>	$\begin{array}{r} 12 \\ \times 5 \\ \hline \end{array}$ <p>Blackline F 23</p>

Fact Fluency with 5's Flashcards, page 2 of 2



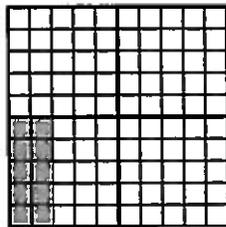
$$20 \div 5$$

Blackline F 24



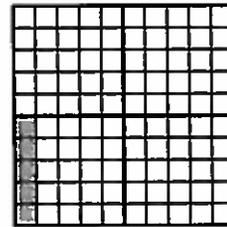
$$15 \div 5$$

Blackline F 24



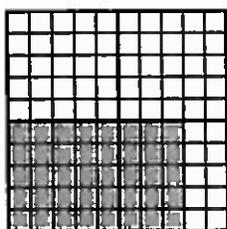
$$10 \div 5$$

Blackline F 24



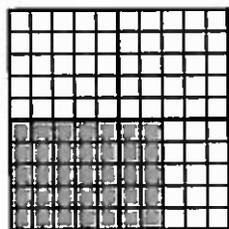
$$5 \div 5$$

Blackline F 24



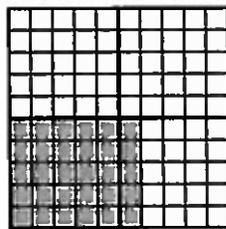
$$40 \div 5$$

Blackline F 24



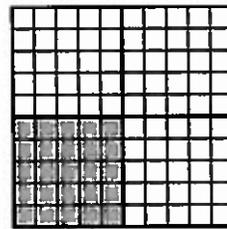
$$35 \div 5$$

Blackline F 24



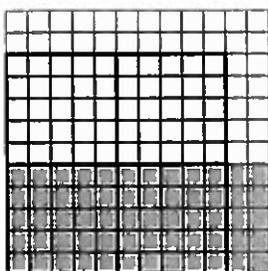
$$30 \div 5$$

Blackline F 24



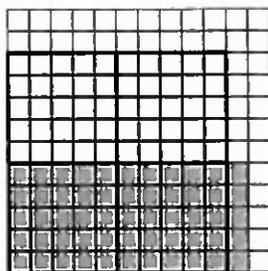
$$25 \div 5$$

Blackline F 24



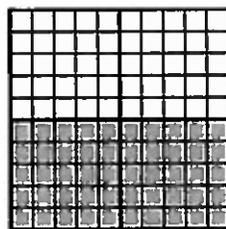
$$60 \div 5$$

Blackline F 24



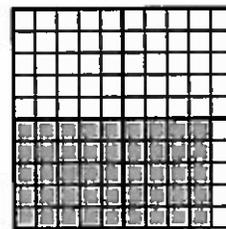
$$55 \div 5$$

Blackline F 24



$$50 \div 5$$

Blackline F 24



$$45 \div 5$$

Blackline F 24

Problem-Solving Guided Math Lessons

Most, if not all, mathematics can best be taught through problem solving. Tasks or problems are posed to engage student in thinking about and developing the important mathematics they need to learn.

The goals of a guided math problem-solving session are to:

- Focus the chosen students' attention on mathematical ideas and sense making
- Develop students' confidence in their capacity to understand mathematics
- Give a context that will assist students in building meaning for a mathematical concept and apply math skills
- Allow multiple entry points to address/support the range of student thinking in your small group setting
- Provide the teacher with ongoing assessment data that is useful for making future instructional decisions that support student success

Football Score

The Warriors scored 42 points and won the football game. How many points did the Mustangs score?

It is a two-digit number.

It is a multiple of 4 and a multiple of 6.

The product of the digits is 18.

The Mustang's score was_____.

Show your work in your journal.

What strategy did you use?

What tip would you share to make this problem easier?

.....

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Show your work in your journal.

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What tip would you share to make this problem easier?

Hannah's Problem

Hannah's age this year is a multiple of 5. Next year, Hannah's age will be a multiple of 4. How old is Hannah now?

Justify your answer. How do you know it is correct? Are there any other correct answers?

Hannah's Problem

Hannah's age this year is a multiple of 5. Next year, Hannah's age will be a multiple of 4. How old is Hannah now?

Justify your answer. How do you know it is correct? Are there any other correct answers?

Stacking Dice: Is it Magical or Mathematical?

Teacher note: You will need 3 standard dice for this activity. The dots on opposite sides of standard six-sided dice always add up to 7, so the sum of the three stacked dice will be 3×7 or 21. Most children and some adults have never explored the relationships of the faces on dice making this “look like magic”.

Begin by gathering data about what the students know about dice. Use the students’ language to discuss what they notice. (Note: students may call the faces ‘sides’, and since the focus of lesson is to use logical reasoning to find the mathematical relationships of the digits in a short period of time, it is important to use their language.)

Roll three dice and quickly stack them on top of each other, glancing quickly at the top face and bottom face in the stack by rotating the stack as you pick them up.

Then quickly determine the sum of the top and bottom faces and subtract that sum from 21 (which is the total sum of all six faces including the four that you cannot see.) The hidden faces will equal that amount. (Example: If the sum of the dots on the top and bottom dice in the stack equals 7, the sum of the hidden faces equals 14.) Tell students the amount of the hidden faces. Then reveal each face, asking students to add the number of dots. Check to see that total equals your “magic” answer.

You might ask students:

- How did I do that?
- What do you know about the dots on a dice?
- Are there any patterns or relationships? That is what a mathematician would be searching for.

Have each student take a die to look at and then talk with a partner about what they notice. Listen and coach the partners as they discuss. After a short discussion, bring students back together to share their ideas with the group.

If the students are not seeing relationships between the opposite faces, you can pick up one die, roll it and close your eyes while placing it on your forehead. Predict what the opposite face of the dice will be. This encourages the children to further explore all the faces of the die and discuss the relationships that they are noticing.

When the students discover that opposite faces of a die will always equal 7, roll all three dice and stack them, just like you did before. Guide students in determining the sum of the 6 faces ($3 \times 7 = 21$). Ask them to add the dots showing on the top and bottom of the stack. Ask the students how they can use these two numbers to determine the sum of the hidden faces. Prompt them by reminding them what they have noticed about the relationships between opposite faces on a die to figure out how to determine the sum of the hidden faces. Have students think, pair, and share, coaching them towards understanding that knowing the total of 6 faces on the three dice equals 21 enables them to subtract the sum of the two visible faces to determine how many dots are on the faces they can’t see ($3 \times 7 - (x+y) = n$).

In a short guided math session, students may not “get” the concept. That’s okay. Simply stop at a certain point for “partial thinking” and have a short discussion about the patterns and relationships students did discover. You can always come back to this same activity for future guided math lessons, building on what students know to bring them to new learning.

For very knowledgeable students who quickly grasp the concept, you might challenge them to explore “magic dice” with different dice that have more than 6 sides to see if the same patterns and relationships exist or discover new patterns.

Resources

- Blanke, B. L. (2004). *Bridges in Mathematics: Number Corner, Grade 3*. Salem, Oregon: The Math Learning Center. www.mathlearningcenter.org. 1 800 575-8130.
- Boaler, Jo. (2008) *What's Math Got to Do With It? Helping Children Learn to Love their Least Favorite Subject—And Why It's Important for America*. New York: Penguin Group.
- Burk, D., & Snider, A. (2002). *Bridges in Mathematics K-2*. Salem, Oregon: The Math Learning Center. www.mathlearningcenter.org. 1 800 575-8130.
- Deerwater, R., Fischer, A. E., & Fisher, A. (2006). *Bridges in Mathematics, Grade 4 & Grade 5*. Salem, Oregon: The Math Learning Center. www.mathlearningcenter.org. 1 800 575-8130.
- Feldman, J. *Songs, Chants and Cheers*. Available on Website: www.drjean.org.
- Foreman, L. C. (1995). *Starting Points for Implementing Visual Mathematics*. Salem, OR: The Math Learning Center, www.mathlearningcenter.org. 1 800575-8130.
- Fosnot, C.T. (2007) *Contexts for Learning Mathematics*. Portsmouth, NH: Heinemann, www.heinemann.com.
- Fosnot, C. T., & Dolk, M. (2001). *Young Mathematicians at Work: Constructing Multiplication and Division*. Portsmouth, NH: Heinemann. www.heinemann.com.
- Fosnot, C. T., & Dolk, M. (2002). *Young Mathematicians at Work: Constructing Fractions, Decimals, and Percents*. Portsmouth, NH: Heinemann. www.heinemann.com.
- Gawronski, J. D. (2005). *Mathematics Assessment Sampler, Grades 3 -5*. Reston, VA: National Council of Teachers of Mathematics. www.nctm.org.
- Hansen-Powell, P. (2004). *Bridges in Mathematics, Grade 3*. Salem, Oregon: The Math Learning Center. www.mathlearningcenter.org.
- Head, D., & Pollett, L. (2005). *Practice Pages. Fractions, Decimals & Percents; Telling Time; Equivalents; Counting Money & Making Change; Elapsed Time*. Shelbyville, KY: bby Publications, www.bbypublications.com. 502 633-9400.
- Head, D., Pollett, L., & Arcidiacono, M. J. (1991). *Opening Eyes to Mathematics*. Salem, Oregon: The Math Learning Center. www.mathlearningcenter.org. 1 800 575-8130.
- Higgins, K. M., Price-Stone, D., & McFadden, S. (1997). *Math Warm Ups*. New York: Dale Seymour Publications, 1 800 321-3106.
- Jensen, E. (1995). *Super Teaching*. San Diego: The Brain Store. www.thebrainstore.com.
- Kagan, Spencer. (2009). *Kagan Cooperative Learning*. www.kaganonline.com.
- Leinwand, Steven., (2009) *Accessible Mathematics*. Portsmouth, NH: Heinemann. www.heinemann.com.

- Marzano, R.I., Pickering, & Pollack. (2001). *Classroom Instruction That Works*. ASCD. <http://www.ascd.org>.
- O'Connell, S. (2007). *Introduction to Problem Solving: Grades 3-5* Portsmouth, NH: Heinemann. www.heinemann.com.
- O'Connell, S. (2005). *Now I Get It: Strategies for Building Confident and Competent mathematicians, K-6*. Portsmouth, NH: Heinemann. www.heinemann.com.
- Sammons, Laney. (2010). *Guided Math: A Framework for Mathematics Instruction*. Huntington Beach, CA: Shell Education. www.shelleducation.com.
- Seely, Cathy L. (2009). *Faster Isn't Smarter*. Math Solutions. www.mathsolutions.com.
- Small, Marion. (2009). *Good Questions: Great Ways to Differentiate Mathematics Instruction*. New York: Teacher's College Press. www.tcp.com.
- Tang, G. (2001). *The Grapes a/Math*. New York: Scholastic Inc.
- Van de Walle, J. A., Karp, Karen & Bay-Williams, Jennifer M. (2010). *Elementary and Middle School Mathematics: Teaching Developmentally* (7th ed.). New York: Allyn & Bacon. www.ablongman.com.
- Van de Walle, J. A., & Lovin, L. H. (2006). *Teaching Student-Centered Mathematics Grades 3-5 & Grades 5-8*. New York: Pearson. www.ablongman.com.

Links to Math Learning Centers and Additional Handouts

Link to general materials similar to those used in the video program:

<<http://mathlearningcenter.org/development/workshops.asp>>

Computational Fluency Materials by grade level:

<<http://mathlearningcenter.org/products/search-advanced.asp>>

Type in Computational Fluency in the search box. That will bring up all the links to each grade level. You will still need to call 1 800 575-8130 to purchase.

Grade 1 - <<http://mathlearningcenter.org/store/product-31694086.htm>>

Grade 2 - <<http://mathlearningcenter.org/store/product-31694088.htm>>

Grade 3: <<http://mathlearningcenter.org/store/product-31694090.htm>>

Grade 4: <<http://mathlearningcenter.org/store/product-31694092.htm>>

Grade 5: <<http://mathlearningcenter.org/store/product-31694094.htm>>

Math Learning Centers to Purchase by Grade Level

<http://mathlearningcenter.org/resources/materials/kindergarten.asp>

<http://mathlearningcenter.org/resources/materials/grade-one.asp>

<http://mathlearningcenter.org/resources/materials/grade-two.asp>

<http://mathlearningcenter.org/resources/materials/grade-three.asp>

<http://mathlearningcenter.org/resources/materials/grade-four.asp>

<http://mathlearningcenter.org/resources/materials/grade-five.asp>

Strategy Books/Journals for FREE download (Multiplication, Addition, Subtraction)

http://mathlearningcenter.org/media/Bridges_Gr3_Corrections/B3TG-B_MultFactsBK.pdf

http://mathlearningcenter.org/media/Bridges_Gr3_Corrections/B3TG-B_AddFactsBK.pdf

http://mathlearningcenter.org/media/Bridges_Gr3_Corrections/B3TG-B_SubFactsBK.pdf

FREE Math Learning Center Directions/WorkPlace Instructions:

Grade 3 WPL instructions:

http://mathlearningcenter.org/files/media/Bridges_Gr3-5_WkPlce_Direction/Bridges_Gr3_WP_Directions.pdf

Grade 4 WPL instructions:

http://mathlearningcenter.org/files/media/Bridges_Gr3-5_WkPlce_Direction/Bridges_Gr4_WP_Directions.pdf

Grade 5 WPL instructions:

http://mathlearningcenter.org/media/Bridges_Gr3-5_WkPlce_Direction/Bridges_Gr5_WP_Directions.pdf

“Not to Miss” Mathematics Websites

<http://nlvm.usu.edu/en/nav/vlibrary.html> – Try this Web site or put “National Library of Virtual Manipulatives for Interactive Mathematics” in Google for other sites. Long name. Cool site! Pre-K-12 students can “play” with these interactive, virtual manipulatives based on the National Math Standards.

www.eduplace.com/math/brain – Great for problem solving! Once a week it provides three problems, one at Gr. 3/4, one at Gr. 5/6, and one at Gr. 7/8. Includes “Getting Started” hints.

<http://abcnews.go.com/Technology/WhosCounting> – John Allen Paulos, the mathematician who wrote *INNUMERACY*, a NY Times bestseller, examines the news through numbers. Best for Gr. 6/7/8.

www.usatoday.com/news/snapshot.htm – In USA TODAY there are four graphs everyday. This Web site provides about 40 such graphs on a rotating basis. It's a wonderful source for math problem solving, graphing, as well as thinking and literacy. Excellent for 4th grade and up.

www.qualityteacherdevelopment.com – An outstanding Web site provided by Sue O'Connell. She has many good math Web sites listed and annotated here. Thanks to Sue for this valuable resource! The following four Web sites are linked to Sue's Web site for easy access:

www.illuminations.nctm.org – Visit the NCTM's Web site for lesson plans and activity ideas at all grade levels.

www.math.com/teachers.html – This site offers lesson plans, classroom resources, links to "free stuff" problems of the week, worksheet generators, and online tutorial assistance.

www.eduplace.com – This Houghton Mifflin Education Place Web site contains brain teasers for grades 3-8.

www.olemiss.edu/mathed/brain/ – This problem of the week Web site sponsored by the University of Mississippi has elementary, middle school, algebra, and geometry problems and solutions.

<http://incompetech.com/graphpaper/plain> – Print any size graph paper for FREE!